

Vibro-agitation of chambered magma

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Abstract

We present the results of a novel set of calculations into the effect of *in situ* pressure reduction of a crystal-rich, basaltic magma layer by propagating seismic (P) waves. Three stages in the process are identified. Critically, an instability can arise such that a low pressure melt layer develops close to the floor in initially densely packed magma (with mean crystal volume fraction $\bar{\phi} = 0.6$) on near-instantaneous timescales. The role of particle pressure, $\bar{p}(z)$, a newly identified force arising from interactions between adjacent crystals in the magma, is fundamental to the development of the instability, which *will not* arise in crystal-free liquids. Key variables governing the instability are identified and include the mean particle diameter, the excitation frequency (1–10 Hz), interstitial melt viscosity and melt compressibility. The quasi-static particle pressure that develops as a result of the spatially-decaying oscillations leads to two effects: (1) a rapid reduction in interstitial melt pressure (c. 10 to 20% ambient pressure) leading to bubble formation, and (2) contraction of a thin magma layer at the base of the magma chamber. Where the basalt layer is overlain by silicic magma, the resulting gas phase may promote local gravitational interaction leading to chamber unrest. The proposed mechanism has implications for the timescales of crystal–liquid separation in magmas, which during the seismic event could be on the order of seconds.

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1. Introduction

This study is concerned with establishing the conditions under which magma stored in the upper crust will exhibit interstitial melt pressure reduction and possible intergranular stress reduction associated with earthquake activity, resulting in unrest and possible eruption. For good reason, most current models of explosive eruption dynamics focus on open-system processes, in particular the underplating of silicic reservoirs by hotter, more mafic magma, as documented recently at Montserrat (Murphy et al., 2000). Theoretical and experimental modelling of

the shallow-level processes governing the eruptive behaviour of dacitic to rhyolitic magmas (e.g. Blake, 1984; Tait et al., 1989; Mader et al., 1994; Woods, 1995), are now relatively sophisticated, and progress into this highly non-linear phenomenon is being made all the time (Melnik and Sparks, 2002; Lautze and Houghton, 2005).

However, it is well known that volcanoes and calderas, including large volume dacite and rhyolitic ignimbrites with eruptive volumes in excess of 1000 km³ (Hildreth, 1981; Lipman, 1984; Christiansen, 2001) develop preferentially in regions that are tectonically active (above subduction zones and hotspots). Where they have been studied in detail, for example the Land and Fish Canyon Tuffs, western United States, and the Cerro Galan and Atana ignimbrites, Andes (De Silva and Francis, 1991; Lindsay et al., 2001), variations in crystallinity and

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chemical zonation trends imply complex pre-eruption histories (Druitt and Bacon, 1989; Reubi and Nicholls, 2005). But despite the potential hazard posed by such large eruptions, the mechanism(s) resulting in their emplacement at the earth's surface remain controversial and poorly understood. As shown repeatedly by instrumental records, seismicity and volcanic unrest are intimately related in continental arcs (Newhall and Dzurisin, 1988). In an extensive survey of caldera unrest, these authors showed that most seismic events in volcanic centres are small discrete earthquakes, with magnitudes (M) typically $3 < M < 6$. The idea that seismicity can trigger eruptions has been put forward by Linde and Sacks (1998) and the evidence reviewed more recently by Hill et al. (2002) and McNutt (2005). The idea of coupling seismology with magma fluid dynamics was highlighted recently by Brodsky et al. (1988). While it is not our intention to promote seismicity above other potential triggers, we nonetheless find it inconceivable, given their tectonic setting, that crustal magma chambers in varying states of crystallinity have not interacted mechanically with seismic waves at some level during their lifetime (see Pritchard and Simons, 2004), possibly with catastrophic effect.

In this article we examine the effects of earthquake activity on the stability of densely packed basaltic magma in a region close to the floor of a layered mafic–silicic magma system. We have in mind a large, partially molten zone in the upper to mid crust, consistent with recent seismic tomographic studies from Yellowstone and Long Valley (Weiland et al., 1995; Miller and Smith, 1999), although our results are equally applicable to magma bodies located at shallower crustal depths. Magma composition is quantified by our choice of viscosity of the interstitial melt phase. A key goal of our analysis is to determine the conditions and governing time scales under which vibro-agitation and potential magma fluidization due to shaking (as opposed to gas streaming) will occur, the latter already recognised as an important process during the eruptive phase (Sparks, 1976; Wilson, 1984).

Consolidation phenomena and constitutive behaviour that take place in soil–fluid–gas mixtures during an externally applied pressure drop have also been studied, although again at near surface conditions only (e.g. Terzaghi, 1943; Theunissen, 1982; Köhler and Koenders, 2003). Importantly, the variables governing fluidization in low pressure water–soil mixtures, including gas solubility, initial saturation, porosity, permeability, particle density, actual and initial fluid pressure are also critically relevant to magmatic systems at higher ambient pressures and temperatures. Using appropriate values, we derive a set of equations that estimate the

change in pressure in the interstitial melt phase due to seismically-induced agitation, located near the bottom of the chamber caused by irreversible grain–grain interactions, and identify the relevant time and length scales. We show how the effect, once initiated, can result in rapid *in situ* bubble formation, and produce abrupt changes in local crystal–melt geometry. The role of compressibility and viscosity of the melt phase are highlighted and we show the instability is strongest where the interstitial liquid viscosity is high (> 10 Pa s), and the grain size (and hence permeability) low. Under these conditions, it is possible that the seismically triggered event is overtaken by a self-sustaining runaway effect (positive feedback), especially for gas production.

2. Background to the problem

The description of two-phase continuous media is a well-established chapter in continuum mechanics, which has applications in various branches of civil and chemical engineering (Koenders et al., 1993; Gidaspow, 1994), but which has also received attention from the physics community (Biot, 1941), and from geologists (Richter and McKenzie, 1984; Bergantz and Ni, 1999; Marsh, 2002; Petford and Koenders, 2003; Bachmann and Bergantz, 2006). The basics may be found in the relevant textbooks, for example Bird et al. (1960). Here the theory is specialised towards particle–fluid mixtures, including a further refinement that pertains to a minority third phase species: a small amount of gas in the fluid (melt). The theory will be applied to describe the mechanics of a layer of melt-embedded grains, agitated by oscillatory loading that originates from the base of the layer. This is a model for a shallow magma chamber with part-granular content that is excited by a P-wave associated with a deep earthquake (Fig. 1).

The mechanics of the problem has two aspects: the first is the actual (high frequency) oscillatory behaviour of the excitation. Various authors have already been concerned with this in a variety of contributions, for example Mikha-lyov (2005). The second part consists of the secular (slow) quasi-static mechanical consequences of the motion, which – while mathematically simple – has not attracted much attention at all. Yet, it is in the quasi-static progression of the problem that the critical stability aspects of the process are found. This should be of great concern for volcanologists. The key to the progression of internal change in the packed granular magma is the development of (1) a ‘particle pressure’ and (2) a reduction of the fluid pressure, both caused by the oscillatory motion of the crystals in the fluid. Both phenomena are localised in an area at the bottom of the chamber where the gradient

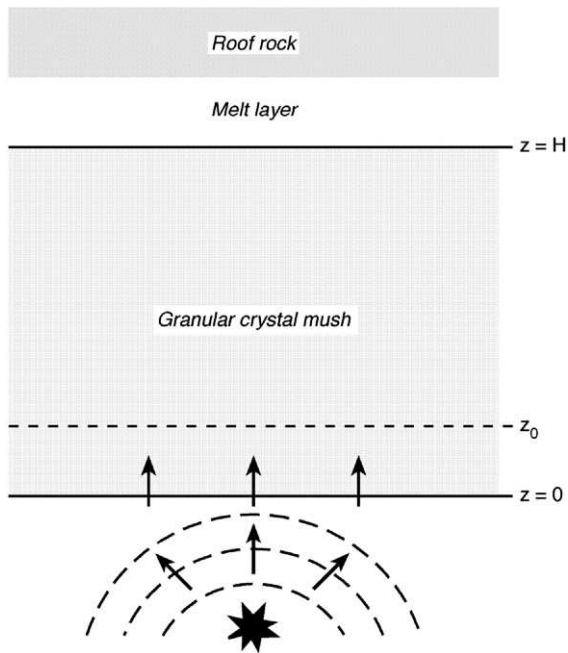


Fig. 1. Schematic showing the geometry of the static, pre-eruptive magma chamber with 50–70% crystal volume fraction on average. The magma has an intrinsic permeability (k), porosity (n), crystal volume fraction (ϕ), stiffness (E), interstitial melt viscosity (η), mean particle size (a) and density contrast $\Delta\rho$ (see Table 1 for values used). The chamber thickness is H . An earthquake located beneath the chamber promotes particle–particle interactions close to the base of the chamber ($z=0$).

of the oscillation amplitude is largest (Fig. 1). While the particle pressure is in all likelihood not sufficient to fluidize the medium, it does cause a temporary change in the stress field, which may be sufficient to trigger plastic deformations in cases where the system as a whole is close to structural instability. A large effect is the lowering of the secular fluid pressure, which may lead to the formation of isolated gas bubbles. These in turn will make the fluid phase more compressible, thus creating a further reduction in the pore pressure, which will create more gas. It is emphasised that these effects are related crucially to the presence of incompressible, hard crystals in the magma.

The effects of agitated slurries have been observed in the laboratory. In small-scale dead-end filtration experiments with vibrated septum a critical point is observed. At this point the particle pressure is sufficiently large to lift the entire bed, clearing the septum and causing a sudden increase in the flow of the filter. The description of this phenomenon – both experimentally and theoretically – is reported in Gundogdu et al. (2003). The theory has been developed further and is reported in Davis and Koenders (in press). Below, the key aspects of concepts involved will be reviewed. The crucial difference

between the experiments reported in Gundogdu et al. (2003) and the magmatic system is that there is no mean vertical flow that assists gravity to stabilise the layer. In addition, the density contrast between melt and crystallised particles is rather smaller than that usually assumed for particles and fluid used in chemical engineering applications. As a result, the magma/crystal system is never very far from instability. *In situ* experimental verification of oscillated slurry effects is naturally not so easy. However, it should be possible to do scaled laboratory experiments similar to the ones that have been done by Gundogdu et al. (2003). The equations of the system should give a good idea of the scaling that ought to be introduced in order for realism to be preserved.

3. Theory

The theory of agitated slurries is treated in detail in a recent paper by Davis and Koenders (in press). Below the relevant equations are summarised and the underlying physical principles explained.

3.1. Governing equations

The particle-phase mass density is denoted by ρ_s ; the crystal volume fraction is ϕ and the solid phase velocity is $\mathbf{v}$. The fluid mass density is called ρ_f and the fluid-phase velocity $\mathbf{u}$. All these are field variables and depend on the position $\mathbf{x}$ and the time t . The basic continuity equations for the solid phase and fluid phase are given in the literature, see Green and Naghdi (1969), Willis (1983), Drew (1986):

$$\frac{\partial(\rho_s \phi)}{\partial t} + \frac{\partial(\rho_s v_i \phi)}{\partial x_i} = 0; \quad (1)$$

$$\frac{\partial}{\partial t} [\rho_f (1 - \phi)] + \frac{\partial[(1 - \phi) \rho_f u_i]}{\partial x_i} = 0, \quad (2)$$

where Einstein's summation convention is used.

The solids mass density is assumed to be constant (Eq. (1) may therefore be divided by ρ_s), but the fluid is compressible and consequently ρ_f may depend on position and time. The main cause of the compressibility is the presence of a small amount of gas in bubble form. The gas phase is a small minority species in the fluid phase. No separate equation set for this phase is necessary as the motion of the gas phase does not significantly affect the stresses in the medium. Its presence is denoted by making the fluid phase compressible. If the compressibility of the fluid is called

β , the volume change of this species is related to the fluid pressure variation and Eq. (2) becomes:

$$(1 - \phi)\beta \frac{Dp}{Dt} - \frac{\partial \phi}{\partial t} + \frac{\partial[(1 - \phi)u_i]}{\partial x_i} = 0, \quad (3)$$

where the substantive derivative D/Dt has been introduced. The compressibility β is introduced such that the relative density change is related to the change in fluid pressure p through $\rho_f^{-1} D\rho_f/Dt = \beta Dp/Dt$.

The equations of motion (the momentum balance equations) are required. These are well-researched and reported in the literature: Green and Naghdi (1969), Willis (1983), Drew (1986). The partial particle phase and fluid (melt) phase stresses are denoted by $\phi\Sigma$ and $(1 - \phi)\sigma$. For the fluid stress a simple isotropic form is assumed: $-p\delta$, where δ is the unity tensor. The intergranular stress τ is obtained from the particle phase stress and the fluid pressure: $\tau = \phi(\Sigma + p\delta)$. The fluid drag force is proportional to the velocity difference in particle and fluid phases with proportionality (drag) coefficient $R(\phi)$. A set of terms is introduced to compensate for the attached mass of fluid to the particles. These terms require a proportionality constant χ . Body forces associated with gravity are also introduced; the acceleration due to gravity is $\mathbf{g}$. For each Cartesian component ($\ell = 1, 2, 3$), the equations read:

$$\begin{aligned} \phi \rho_s \frac{Dv_\ell}{Dt} = & \frac{\partial}{\partial x_i} (\phi \Sigma_{\ell i}) + \phi R(\phi)(u_\ell - v_\ell) \\ & + \chi \phi \rho_f \frac{D}{Dt} (u_\ell - v_\ell) + \phi \rho_s g_\ell; \end{aligned} \quad (4)$$

$$\begin{aligned} (1 - \phi) \rho_f \frac{Du_\ell}{Dt} = & \frac{\partial}{\partial x_i} ((1 - \phi) \sigma_{\ell i}) - \phi R(\phi)(u_\ell - v_\ell) \\ & + -\chi \phi \rho_f \frac{D}{Dt} (u_\ell - v_\ell) \\ & + (1 - \phi) \rho_f g_\ell. \end{aligned} \quad (5)$$

There are a number of further small corrective terms reported in the literature, see for example Drew (1986). These are all neglected for the problem in hand.

Instead of the fluid and solids velocity fields $\mathbf{u}$ and $\mathbf{v}$ it is both advantageous and physically more meaningful to work with the discharge (superficial) fields $\mathbf{q} \equiv (1 - \phi) \mathbf{u}$ and $\mathbf{r} \equiv \phi \mathbf{v}$. This is further elaborated in Davis and Koenders (in press). Closure of the equations is achieved in the usual way by introducing a stiffness E and a damping γ , relating the particle-phase stress to strain ϵ and strain-rate $\dot{\epsilon}$ respectively.

Below two time-scales are distinguished. These are the fast, oscillating time scale, which is explored for small fluctuations, and the secular, quasi-static changes in the packed bed, which accompany the oscillations. The two are coupled: the high-frequency variations (with circular frequency ω) lead to secular effects and the slow changes lead to a modified response in the fast variations. One of the effects associated with fast fluctuations is the manifestation of a more or less constant *particle pressure*; this effect is described in the literature and is associated with inelastic collisions or hysteretic phenomena, see for example McTigue and Jenkins (1992). It is also reviewed in the paper by Davis and Koenders (in press). Oscillatory solutions of the equations of motion are obtained by linearisation and consideration of the dominant Fourier component only. Non-trivial correlations between out-of-phase field variables lead to secular terms, which will be incorporated into the quasi-static equations. All field parameters are written as a time-averaged part (denoted by an over bar) plus an oscillating part (indicated by a tilde); for example $\phi(x, t) = \bar{\phi}(x, t) + \tilde{\phi}(x, t)$. The particle pressure, while induced by oscillations, is quasi-static in nature and denoted by $\bar{\pi}$. Its value due to collisions in a vigorously agitated system has been estimated by Gundogdu et al. (2003). The momentum transfer that takes place during collisions depends on the interparticle force present before the collision. This force scales according to the lubrication force in squeeze flow as particles approach or depart. The scaling is proportional to the fluid viscosity η and the ratio of the particle diameter to the mean gap width a/h . The magnitude of the relative velocity difference between particles is estimated from the amplitude of the velocity gradient $\partial \tilde{v}/\partial z$. Thus Gundogdu et al. (2003) obtain:

$$\bar{\pi} = c\eta \left| \frac{\partial \tilde{v}}{\partial z} \right| \frac{a}{h}, \quad (6)$$

where c is a constant of the order of 0.3–0.8. This estimate of the particle pressure is entirely based on the slurry properties of the particle–fluid mixture. For densely packed media with enduring contacts other terms come into play as well, increasing the particle pressure from the value estimated by expression (19).

The equations that describe the secular change in the oscillated, packed magma slurry are derived by setting all dynamic terms equal to zero. The quasi-static evolution in the magma is a small variation superimposed on static equilibrium. It is caused by the ‘switching on’ of the particle pressure. There are thus three stages in the process: (1) the stage before the earthquake starts —

the static bed with no additional particle pressure; (2) the secular evolution after the earthquake has excited the bed and the particle pressure is first manifest; (3) the stage that describes the stresses *etc* a long time after the earthquake has started. What is important here is the time constants of the problem, that is, the parameters of the problem itself indicate what is meant by ‘a long time’. Typically, the period of oscillation is of the order of 0.1 s. It will be shown below that the time constant of the secular change is of the order of 1–10 s and the duration of the quake is approximately 60–100 s. With these estimates it is clear that the effects of the excitation will be fully consolidated in the lifetime of the quake.

3.2. Static case (stage 1)

First, the purely static limit is considered (denoted by subscript *stat*). All the velocities are zero and all time-dependent terms are irrelevant. The stresses depend on the vertical coordinate z only and they obey the set:

$$\frac{\partial \bar{\tau}_{\text{stat}}}{\partial z} - \bar{\phi} \frac{\partial \bar{p}_{\text{stat}}}{\partial z} - \bar{\phi} \rho_s g = 0 \quad (7)$$

$$-\frac{\partial \bar{p}_{\text{stat}}}{\partial z} - \rho_f g = 0 \quad (8)$$

These equations are of course easily solved. At the top of the layer of sediment ($z=H$) the intergranular stress vanishes; the fluid pressure is the full pressure at the relevant depth $\bar{p}_{\text{stat}}(0)$ and therefore:

$$\bar{\tau}_{\text{stat}} = \bar{\phi}(\rho_s - \rho_f)g(z - H) \quad (9)$$

$$\bar{p}_{\text{stat}} = \bar{p}_{\text{stat}}(0) - \rho_f g z \quad (10)$$

The static pressures define the parameter range of the problem. Typically $\bar{p}_{\text{stat}}(0)$ is of the order of 10^7 – 10^8 Pa. The sediment layer has a thickness of some 10–100 m; given the small density contrast between solids and fluids the grains are therefore only very lightly loaded — comparable with shallow soil mechanical problems.

3.3. Oscillatory solutions (stage 2)

When the Eqs. (1), (2), (4) and (5) are expanded in terms of the oscillatory field variables, the parameters E and R must be expanded separately. So, for example, the term $\phi R(\phi)(u_\ell - v_\ell) = \phi R(\phi)(q_\ell/(1 - \phi) - r_\ell/\phi)$ has a variation $R(\phi)(\bar{\phi}\tilde{q}_\ell/(1 - \bar{\phi}) - \tilde{r}_\ell + \bar{\phi}\tilde{q}_\ell/(1 - \bar{\phi})^2) + \partial R/\partial \phi (\bar{\phi}\tilde{q}_\ell/(1 - \bar{\phi}) - \tilde{r}_\ell)\tilde{\phi}$, which contains the sensi-

tivity of the drag coefficient to crystal volume fraction variations $\partial R/\partial \phi$.

Simplifying assumptions will be made. The first of these is that

1. The packed bed and the interstitial fluid are on average at rest and therefore $\bar{r}$ and $\bar{q}$ can both be set to zero. The equations that describe the oscillated slurry are then derived by Davis and Koenders (in press) and take the form:

$$\frac{\partial \tilde{\phi}}{\partial t} + \frac{\partial \tilde{r}_z}{\partial z} = 0; \quad (11a)$$

$$(1 - \bar{\phi})\beta \frac{D\tilde{p}}{Dt} - \frac{\partial \tilde{\phi}}{\partial t} + \frac{\partial \tilde{q}_z}{\partial z} = 0; \quad (11b)$$

$$\begin{aligned} \rho_s \frac{\partial \tilde{r}_z}{\partial t} = & \frac{\partial}{\partial z} (\overline{E(\phi)} \tilde{e}_{zz} + \overline{\gamma(\phi)} \frac{\partial}{\partial z} \left(\frac{\tilde{r}_z}{\bar{\phi}} \right) \\ & - \bar{\phi} \tilde{p} - \bar{p} \tilde{\phi} + \overline{R(\phi)} \left(\frac{\bar{\phi} \tilde{q}_z}{1 - \bar{\phi}} - \tilde{r}_z \right) \\ & + \chi \bar{\phi} \rho_f \frac{\partial}{\partial t} \left(\frac{\bar{\phi} \tilde{q}_z}{1 - \bar{\phi}} - \tilde{r}_z \right) - \tilde{\phi} \rho_s g; \end{aligned} \quad (11c)$$

$$\begin{aligned} \rho_f \frac{\partial \tilde{q}_z}{\partial t} = & - \frac{\partial}{\partial z} [(1 - \bar{\phi}) \tilde{p} - \bar{\phi} \tilde{p}] \\ & - \overline{R(\phi)} \left(\frac{\bar{\phi} \tilde{q}_z}{1 - \bar{\phi}} - \tilde{r}_z \right) \\ & - \chi \bar{\phi} \rho_f \frac{\partial}{\partial t} \left(\frac{\bar{\phi} \tilde{q}_z}{1 - \bar{\phi}} - \tilde{r}_z \right) + \tilde{\phi} \rho_f g. \end{aligned} \quad (11d)$$

Davis and Koenders (in press) show that it is reasonable to regard $\bar{p}$ as a constant in this analysis. In that case the solution of the oscillatory field parameters has the form $\exp(\lambda z) \exp(i\omega t)$ and a value for λ is easily derived. The average is evaluated over a length scale of the order of magnitude of $|\lambda|^{-1}$; $1/Re(\lambda)$ is the *penetration depth*. Davis and Koenders (in press) then make a number of assumptions concerning the order of magnitude of the material parameters in the problem. Here different assumptions are made; these are relevant to the high fluid pressure regime and low sediment bed stiffness that

Table 1

Typical values for the parameters of the problem for a magma chamber

ρ_f	$2.5 \times 10^3 \text{ kg m}^{-3}$	ω	60 s^{-1}	H	100 m
ρ_s	$2.7 \times 10^3 \text{ kg m}^{-3}$	E	$5. \times 10^8 \text{ Pa}$	γ	10 η
$\bar{\phi}$	0.6	β	$5. \times 10^{-9} \text{ Pa}^{-1}$	a	10^{-3} m
χ	0.5	η	10 Pa s	g	10 ms^{-2}
$\bar{R}$	$10^{10} \text{ Pa s m}^{-2}$	κ	100	a/h	100
$\bar{p}$	10^8 Pa	λ_0	$1/(10a)$		

may be expected. Estimates of the values of the parameters are given in Table 1.

- (1) The stiffness $\bar{E}$ and the mean fluid pressure $\langle \bar{p} \rangle$ are in the same order of magnitude. This can be determined simply from the numbers.
- (2) The mean crystal volume fraction $\bar{\phi}$ is in the range 0.5–0.7. These are normally expected values for an unfluidised granular medium.
- (3) The parameter $\omega \rho_f \ll R$. The resistance R is proportional to η/a^2 , where η the fluid viscosity and a the particle diameter. Now, the ratio $\omega \rho_f / R$ is proportional to $(\omega a) \rho_f a / \eta$. As (ωa) has the dimension of a speed, the ratio $\omega \rho_f / R$ is akin to a Reynolds number. So, other than doing the numbers for a particular case, it is observed that the oscillatory motion leads to slow interstitial flow.
- (4) $g \rho_f$ is much smaller than $|\lambda| \bar{E}$; this follows simply from a comparison of the numbers for $|\lambda| \sim a^{-1}$. This is an approximation that says something about the nature of the localised phenomenon.

In practical terms the difference between ρ_f and ρ_s is small, so one may set $\rho_s = \rho_f + dp$, where dp/ρ_f is a number that is substantially smaller than unity. It is also observed that in practice $\omega \gamma / \bar{E}$ is a small parameter. Now, making good use of approximation 4, above, it is found that up to first order in dp and $\omega \gamma$ the two roots λ^2 are:

$$\lambda^2 = \frac{i \bar{R} \bar{\phi} \omega (\bar{E} \beta (1 - \bar{\phi}) + 1)}{(1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})} + \frac{d \rho \bar{\phi} (1 - \bar{\phi}) \omega^2 (1 - \bar{\phi})(\beta \bar{p} \bar{\phi} - 1)}{(\bar{E} \beta (1 - \bar{\phi}) + 1)(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})} + \frac{\gamma \omega \bar{E} R \bar{\phi} (1 - \beta \bar{p} \bar{\phi})}{(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})^2} - \frac{\beta d \rho \bar{\phi} \omega^2 (1 - \bar{\phi})}{\bar{E} \beta (1 - \bar{\phi}) + 1}. \quad (12)$$

The largest value (by far) is the first root; the one with the negative real part is chosen. For this case, the relationship between the various field parameters is expressed in the ratios of the amplitudes, which are denoted by hats:

$$\begin{aligned} \hat{p} &= -\frac{\bar{E} \sqrt{2 \bar{R} (\bar{E} \beta (1 - \bar{\phi}) + 1)} (1 - i)}{2 \sqrt{\bar{\phi} \omega (1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})}} \hat{r}; \\ \hat{\phi} &= \frac{\sqrt{2 \bar{R} \bar{\phi} (\bar{E} \beta (1 - \bar{\phi}) + 1)} (1 - i)}{2 \sqrt{\bar{E} \omega (1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})}} \hat{r}; \\ \hat{q} &= \frac{\bar{E} \beta (1 - \bar{\phi}) + \bar{\phi}}{\bar{\phi}} \hat{r}. \end{aligned} \quad (13)$$

Davis and Koenders (in press) also find the means of the cross-products. Defining the inverse penetration depth $Re(\lambda) = \lambda_0 = \frac{1}{2} \times \sqrt{2 \bar{R} \bar{\phi} \omega (\bar{E} \beta (1 - \bar{\phi}) + 1) / [(1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})]}$, the cross product averages that are necessary for the development of the secular equation can be evaluated. For example:

$$\begin{aligned} \overline{\tilde{r}_z \tilde{\phi}} &= \frac{\omega}{2\pi} \int_0^{2\pi/\omega} Re(\hat{r} e^{i\omega t} e^{-\lambda z}) Re(\hat{\phi} e^{i\omega t} e^{-\lambda z}) dt \\ &= \frac{\sqrt{2 \bar{R} \bar{\phi} (\bar{E} \beta (1 - \bar{\phi}) + 1)} e^{-2\lambda_0 z}}{4 \sqrt{\omega (1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})}} \hat{r}^2. \end{aligned} \quad (14)$$

The others are obtained in a similar way:

$$\overline{\tilde{q}_z \tilde{\phi}} = -\frac{\sqrt{2 \bar{R} (\bar{E} \beta (1 - \bar{\phi}) + 1)} (\bar{E} \beta (1 - \bar{\phi}) + \bar{\phi}) e^{-2\lambda_0 z}}{4 \sqrt{\bar{\phi} (1 - \bar{\phi}) \omega (\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})}} \hat{r}^2; \quad (15)$$

$$\overline{\tilde{p} \tilde{\phi}} = -\frac{\bar{E} R (\bar{E} \beta (1 - \bar{\phi}) + 1) e^{-2\lambda_0 z}}{2 \omega (1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})} \hat{r}^2; \quad (16)$$

$$\frac{\partial \tilde{p}}{\partial t} \tilde{\phi} = 0; \quad (17)$$

$$\overline{\tilde{e}_{zz} \tilde{\phi}} = -\frac{\bar{R} (\bar{E} \beta (1 - \bar{\phi}) + 1) e^{-2\lambda_0 z}}{2 \omega (1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p} \bar{\phi})} \hat{r}^2. \quad (18)$$

Finally, the particle pressure becomes:

$$\bar{\pi} = c \eta \left| \frac{\partial \tilde{v}}{\partial z} \right| \frac{a}{h} = c \eta \lambda_0 \frac{a}{h} |\hat{r}| e^{-\lambda_0 z}. \quad (19)$$

3.4. Secular solutions (stage 2)

The secular equations are obtained from Eqs. (1), (2), (4) and (5) by time averaging over one cycle. They involve averages of fields and averages of double products of fluctuations of fields. Two parameters are introduced, C_R defined as $\partial \bar{R} / \partial \bar{\phi}$ and C_E as $\partial \bar{E} / \partial \bar{\phi}$; forward evaluation yields the following:

$$\frac{\partial \bar{\phi}}{\partial t} + \frac{\partial \bar{r}_z}{\partial z} = 0. \quad (20a)$$

$$\begin{aligned} (1 - \bar{\phi}) \beta \left(\frac{\partial \bar{p}}{\partial t} + \frac{\bar{q}_z}{1 - \bar{\phi}} \frac{\partial \bar{p}}{\partial z} \right) - \beta \bar{\phi} \frac{\partial \tilde{p}}{\partial t} - \frac{\partial \bar{\phi}}{\partial t} \\ + \frac{\partial \bar{q}_z}{\partial z} = 0; \end{aligned} \quad (20b)$$

$$\begin{aligned}
& \frac{\partial}{\partial z} (\bar{\tau}_{zz} + \bar{\pi} + C_E \bar{\varepsilon}_{zz} \bar{\phi} - \bar{\phi} \bar{p} - \bar{\phi} \bar{p}) \\
& + \bar{R} \left(\frac{\bar{\phi} \bar{q}_z}{(1 - \bar{\phi})} - \bar{r}_z \right) \\
& + C_R \left(\frac{\bar{\phi} (\bar{\phi} \bar{q}_z)}{1 - \bar{\phi}} - \bar{\phi} \bar{r}_z + \frac{(\bar{\phi}^2) \bar{q}_z}{(1 - \bar{\phi})^2} \right) \\
& - \bar{\phi} \rho_s g = 0; \quad (20c)
\end{aligned}$$

$$\begin{aligned}
& - \frac{\partial}{\partial z} [(1 - \bar{\phi}) \bar{p}] + \frac{\partial}{\partial z} (\bar{\phi} \bar{p}) - \bar{R} \left(\frac{\bar{\phi} \bar{q}_z}{(1 - \bar{\phi})} - \bar{r}_z \right) \\
& - C_R \left(\frac{\bar{\phi} (\bar{\phi} \bar{q}_z)}{1 - \bar{\phi}} - \bar{\phi} \bar{r}_z + \frac{(\bar{\phi}^2) \bar{q}_z}{(1 - \bar{\phi})^2} \right) \\
& - (1 - \bar{\phi}) \rho_f g = 0. \quad (20d)
\end{aligned}$$

The appropriate term for the particle pressure has also been introduced here. The term $\partial \bar{\phi} / \partial t$ is eliminated and the cross products are inserted. These steps lead to the following simple equations:

$$\frac{\partial \bar{\phi}}{\partial t} + \frac{\partial \bar{r}_z}{\partial z} = 0. \quad (21a)$$

$$(1 - \bar{\phi}) \beta \left(\frac{\partial \bar{p}}{\partial t} + \frac{\bar{q}_z}{1 - \bar{\phi}} \frac{\partial \bar{p}}{\partial z} \right) + \frac{\partial \bar{r}_z}{\partial z} + \frac{\partial \bar{q}_z}{\partial z} = 0; \quad (21b)$$

$$\begin{aligned}
& \frac{\partial}{\partial z} (\bar{\tau}_{zz} + \hat{\pi} e^{-\lambda_0 z} - B e^{-2\lambda_0 z} - \bar{\phi} \bar{p}) \\
& + \bar{R} \left(\frac{\bar{\phi} \bar{q}_z}{(1 - \bar{\phi})} - \bar{r}_z \right) - \bar{\phi} \rho_s g = A e^{-2\lambda_0 z}; \quad (21c)
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial z} [(1 - \bar{\phi}) \bar{p}] + \bar{R} \left(\frac{\bar{\phi} \bar{q}_z}{(1 - \bar{\phi})} - \bar{r}_z \right) \\
& - (1 - \bar{\phi}) \rho_f g = A e^{-2\lambda_0 z}, \quad (21d)
\end{aligned}$$

where

$$A = \frac{\sqrt{2\bar{\phi}\bar{R}} (2\bar{E}\bar{R} + C_R(\bar{E}(1 - \bar{\phi}) + \bar{p}\bar{\phi})(\bar{E}\beta(1 - \bar{\phi}) + 1)^{3/2})}{4\sqrt{\omega}(1 - \bar{\phi})^{3/2}(\bar{E}(1 - \bar{\phi}) + \bar{p}\bar{\phi})^{3/2}} \hat{r}^2; \quad (22)$$

$$B = \frac{C_E \bar{R} (\bar{E}\beta(1 - \bar{\phi}) + 1)}{2\omega(1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p}\bar{\phi})} \hat{r}^2; \quad (23)$$

$$\hat{\pi} = c\eta\lambda_0 \frac{a}{h} |\hat{r}|. \quad (24)$$

Some time after the oscillation has been ‘switched on’ the system will be in steady state: the velocities and time

derivatives will be negligible. Then, assuming $H \gg \lambda_0^{-1}$, the solution for the stresses is simply:

$$\begin{aligned}
\bar{\tau}_{zz} = & -\hat{\pi} e^{-\lambda_0 z} + B e^{-2\lambda_0 z} - \frac{A e^{-2\lambda_0 z}}{2\lambda_0(1 - \bar{\phi})} \\
& + g\bar{\phi}(z - H)(\rho_s - \rho_f); \quad (25)
\end{aligned}$$

$$\bar{p} = -\frac{A e^{-2\lambda_0 z}}{2\lambda_0(1 - \bar{\phi})} - g(z - H)\rho_f. \quad (26)$$

The terms are elaborated:

$$\begin{aligned}
B = & \frac{A}{2\lambda_0(1 - \bar{\phi})} = \frac{\bar{R}\bar{E}(\bar{E}\beta(1 - \bar{\phi}) + 1)}{2\omega(1 - \bar{\phi})(\bar{E}(1 - \bar{\phi}) + \bar{p}\bar{\phi})} \\
& \times \left(\frac{C_E}{\bar{E}} - \frac{C_R \bar{p} \bar{\phi}}{2\bar{E}\bar{R}(1 - \bar{\phi})} - \frac{C_R}{2\bar{R}} - \frac{1}{(1 - \bar{\phi})} \right) \hat{r}^2; \quad (27)
\end{aligned}$$

$$\begin{aligned}
-\frac{A}{2\lambda_0(1 - \bar{\phi})} = & -\frac{\bar{R}\bar{E}(\bar{E}\beta(1 - \bar{\phi}) + 1)}{4\omega(1 - \bar{\phi})^2(\bar{E}(1 - \bar{\phi}) + \bar{p}\bar{\phi})} \\
& \times \left(\frac{C_R \bar{p} \bar{\phi}}{\bar{E}\bar{R}} + \frac{C_R(1 - \bar{\phi})}{2\bar{R}} + 2 \right) \hat{r}^2. \quad (28)
\end{aligned}$$

In a separate analysis Davis and Koenders (in press) show that the time constant involved in reaching the steady state is maximally of the order of magnitude of the duration of one cycle of the quake.

4. Results and implications

The results of the theoretical analysis are now explored. Estimates for the parameters in the problem are first provided. The drag coefficient $\bar{R}$ is derived from an estimate Happel and Brenner (1991), and depends on the melt viscosity η and the particle diameter a and is a function of the crystal volume fraction ϕ :

$$\bar{R} = \frac{18\bar{\phi}\eta(1 - \bar{\phi})}{a^2} \frac{3 + 2\sqrt[3]{\bar{\phi}^5}}{3 - \frac{9}{2}(\sqrt[3]{\bar{\phi}} - \sqrt[3]{\bar{\phi}^5}) - 3\bar{\phi}^2} \quad (29)$$

For the rheology of the dynamic stiffness $E(\bar{\phi})$ the literature on oscillated compressed packed beds is useful. According to Hardin and Richart (1963) this modulus (in samples with isotropic prestress τ_{iso}) depends on the void ratio $e = (1 - \bar{\phi})/\bar{\phi}$ and has the form:

$$E = f_0 \tau_{iso}^{1/2} (2.17 - e)^2 / (1 + e), \quad (30)$$

where f_0 depends on the material parameters of the grains. A reasonable value for granular magma is $f_0 = 3 \times 10^4 \text{ Pa}^{1/2}$.

The particle pressure given by expression (19) requires an estimate for a/h . For spherical particles an estimate is available: Torquato et al. (1990):

$$\frac{a}{h} = \frac{6\bar{\phi}(2 - \bar{\phi})}{(1 - \bar{\phi})^3} \quad (31)$$

For non-spherical particles no estimate is available at this stage. For high values of $\bar{\phi}$ the parameter a/h rises fast, indicating that the linear particle pressure becomes important at high densities; this term works so as to expand the packed bed. The quadratic particle pressure – expression (27) – may work either expansively or lead to contraction. With the estimates for E and $\bar{R}$ the quadratic particle pressure consists of a positive front factor and the non-dimensional term:

$$p_s = \left(\frac{C_E}{E} - \frac{C_R \bar{p} \bar{\phi}}{2ER(1 - \bar{\phi})} - \frac{C_R}{2R} - \frac{1}{(1 - \bar{\phi})} \right). \quad (32)$$

p_s depends on $\bar{\phi}$ and the ratio $x_p \equiv \bar{p}/E$. Fig. 2 shows the behaviour. A negative value of p_s implies contraction, a positive one expansion. Thus, as far as the quadratic contribution to the particle pressure is concerned, for low packing fractions the magma layer will expand to lower values, while for densely packed media ($\bar{\phi} > 0.5$), the oscillation will further contract it.

The order of magnitude of the particle pressure is a multifactorial question. Other than the packing fraction the gas content, the frequency of oscillation and the amplitude of the agitation are all involved. Also, the penetration of the

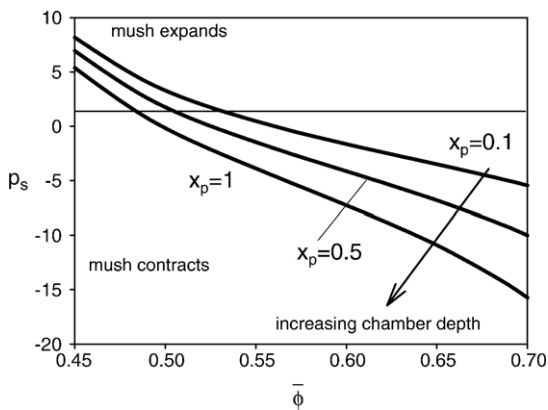


Fig. 2. The proportionality constant p_s , defined in Eq. (32), as a function of the mean crystal volume fraction $\bar{\phi}$ at various values of the ratio of the ambient pressure, a function of magma chamber depth, to the skeletal stiffness of the mush. The figure gives an impression of the induced extra-particle stress due to the quadratic term as a function of increasing magma chamber depth beneath the surface (x_p increasing). The results imply that for magma mushes with $>50\%$ solids, seismic agitation results in contraction.

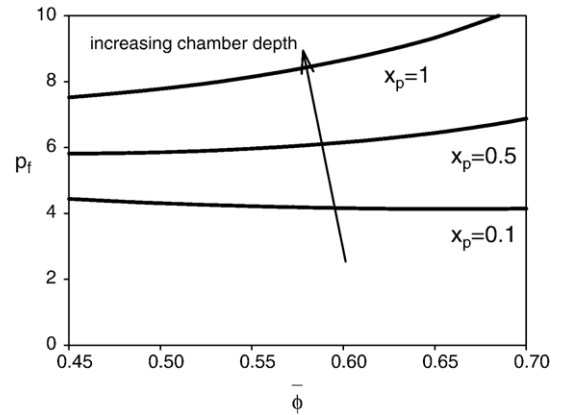


Fig. 3. The proportionality constant p_f defined in Eq. (33), as a function of the mean crystal volume fraction $\bar{\phi}$ at various values of the ratio of the ambient pressure to the skeletal stiffness (see Fig. 2 for definition of terms). The figure gives an impression of the induced extra fluid pressure reduction due to the quadratic term. It is this effect that causes bubbles to form during seismic agitation. Arrow shows effect of increasing depth of magma chamber from the surface. The higher the ambient pressure (deeper the chamber), the greater the rate of pressure reduction and hence bubble formation.

linear effect is less localised (proportional to $\exp(-\lambda_0 z)$) than the quadratic effect (proportional to $\exp(-2\lambda_0 z)$). Some practical values based on the parameter estimates given here indicate that for oscillations with an amplitude of one particle diameter (1 mm) the quadratic effect is far stronger at $z=0$, implying a contraction of the bed. Increased frequency has a profound magnifying effect on all contributions to the particle pressure. Increasing gas content increase the particle pressures slightly. But the key parameter in this is the melt viscosity; it both enlarges the pressure contributions, but also – disproportionately – makes the penetration depth smaller (proportional to $\eta^{-1/2}$), thus increasing the linear particle pressure (proportional to $\eta^{3/2}$) and the quadratic contribution (proportional to η). At high viscosities (order 100 Pa s) all effects are very large, implying accute densification of the magma mush in a thin zone where $\exp(-2\lambda_0 z)$ dominates and severe expansion in a thin zone just above it, where $\exp(-\lambda_0 z)$ is greater than $(-2\lambda_0 z)$. Such locally diverse behaviour is likely to have severe consequences for the overall stability of the magma layer, especially where the chamber is compositionally and/or rheologically zoned (see discussion). Finally, increased gas content has a minor effect on the particle pressure contributions.

In all this the linear theory has been used giving a first idea of the effects; it is very doubtful that the linear theory is correct for large amplitudes when the magma has both solid and fluid properties. To gain more insight in the non-linear properties of the system a discrete computer simulation would have to be done.

The fluid pressure variation under oscillation is of great interest. There is no linear term; the term in brackets in expression (28) is:

$$p_f = \left(\frac{C_R \bar{p} \bar{\phi}}{ER} + \frac{C_R(1 - \bar{\phi})}{2R} + 2 \right), \quad (33)$$

which is plotted in Fig. 3 for various values of the ratio x_p . Because of the minus sign in expression (28) the fluid pressure always decreases. The magnitude of decrease is typically of the order of 5–10% of the static pressure for $\eta = 10$ Pa s (all other parameters as quoted before) and is linear with viscosity, implying that if the magma is highly viscous (dacite–rhyolite), negative pressures would actually result. It is also linear in the frequency of agitation.

The effect of a fast-falling fluid pressure in a localised zone is the generation of gas bubbles. This is the main effect in terms of unrest. The pressure fall-rate is typically of the order of magnitude of 10^6 Pa s^{−1}.

5. Discussion

A number of implications for the dynamics of layered mafic–silicic crustal magma chambers flow from our model. Firstly, the idea of a rheological lock-up point at high crystal contents (e.g. Vigneresse et al., 1996) is meaningless in this context as the excitation mechanism results in disruption of the solid mush at crystal contents in excess of 50%. Indeed, it is the requirement of the granular framework to actively deform at crystal contents in the range $0.45 < \bar{\phi} < 0.7$ that provides the novelty of the mechanism. Fig. 2 gives insight into the behaviour of the oscillating magma layer at high crystal loads. For an ambient magma pressure of 10^7 Pa (equating to a depth of c. 5 km), seismic agitation will result in net contraction of the mush in the basal layer. At the same time, the pressure in the melt phase drops. To understand what might happen in the plutonic environment, we can draw parallels with volcanic processes accompanying decompression as magma ascends towards the surface — e.g. Sparks (1978). In the uppermost crust, low pressures and high magma crystal volume fraction promote degassing and bubble nucleation: Mangan and Sisson (2000). Bubble formation in magma is strongly dependent on the attainment of low pressures, due in part to a large kinetic barrier preventing nucleation directly from the melt phase (Sparks, 1978), although this is ameliorated by heterogeneous nucleation on crystals (Mangan et al., 2004). In their unperturbed, static state, pressures and gas solubilities in magma reservoirs deeper than several km are such that

volatiles will not exolve, despite the presence of crystals (Moore et al., 1995; Wallace, 2001). However, as shown above, during excitation the melt pressure is reduced substantially such that the magma locally undergoes *in situ* decompression. The calculated local pressure drop ΔP over a characteristic time interval (t) of 10 s lies at the upper end of the experimental values (0.025 – 1 MPa s^{−1}) known to promote heterogeneous bubble formation in viscous magmas (Mangan and Sisson, 2000; Rutherford and Gardner, 2000). The fate of these bubbles is worth exploring. Most studies of bubble formation in magmas have focused on open-systems where degassing and magma recharge are intimately linked (e.g. Phillips and Woods, 2001; Bachmann and Bergantz, 2006). A counter-intuitive observation is that during seismic agitation, bubble formation will actually increase with increasing magma chamber depth from the surface. This is because bubble formation is governed by the rate of pressure drop, so that for higher initial ambient pressures (deeper magma chambers), we predict more, smaller sized bubbles. For shallower chambers the model predicts fewer, larger bubbles (Fig. 3).

While our model is effectively a closed system processes, there is nonetheless a striking complementarity with high-level volcanic systems. For example, once formed, gas bubbles may migrate upwards through the magma pile, either gravitationally, or as a result of extremely rapid segregation of melt into transient, high permeability channels (possibly analogous to shrinkage cracks) in the contracting mush. Over time, bubbles could find themselves trapped at the interface between the top of the mafic mush and overlying silicic magma, possibly forming a separate foam layer (e.g. Tait et al., 1989). Subsequent gravitational instabilities here could result in localised mingling of magmas across the mafic–silicic interface in a manner similar to that proposed by Couch et al. (2001) or, following Bachmann and Bergantz (2006), rejuvenation of the overlying silicic magma by gas sparging. The latter model has been proposed as a mechanism for transferring heat upwards from a basaltic layer into more evolved magma without the need for significant chemical interaction. We note in passing that while our proposed mechanism is iso-chemical, the agitation effect will require significant viscous dissipation of heat. The thermal consequences of the model are yet to be explored, but following recent work on viscous dissipation in volcanic flows (Vedeneva et al., 2005), are likely to be important.

In some ways this sequence of events, in particular the role played by bubble formation, is complementary with the advective overpressure and rectified diffusion model of Brodsky et al. (1988), although differs in the

important respect that here it is the particle pressure that is responsible for decompression and bubble formation. Whatever the ultimate fate of the bubbles, changes cited as diagnostic of magma unrest at depth including

harmonic tremor, rapid variations in seismic energy release and surface uplift (Newhall and Dzurisin, 1988), are not incompatible with our model of chamber-wide magma agitation, indeed we consider them highly likely

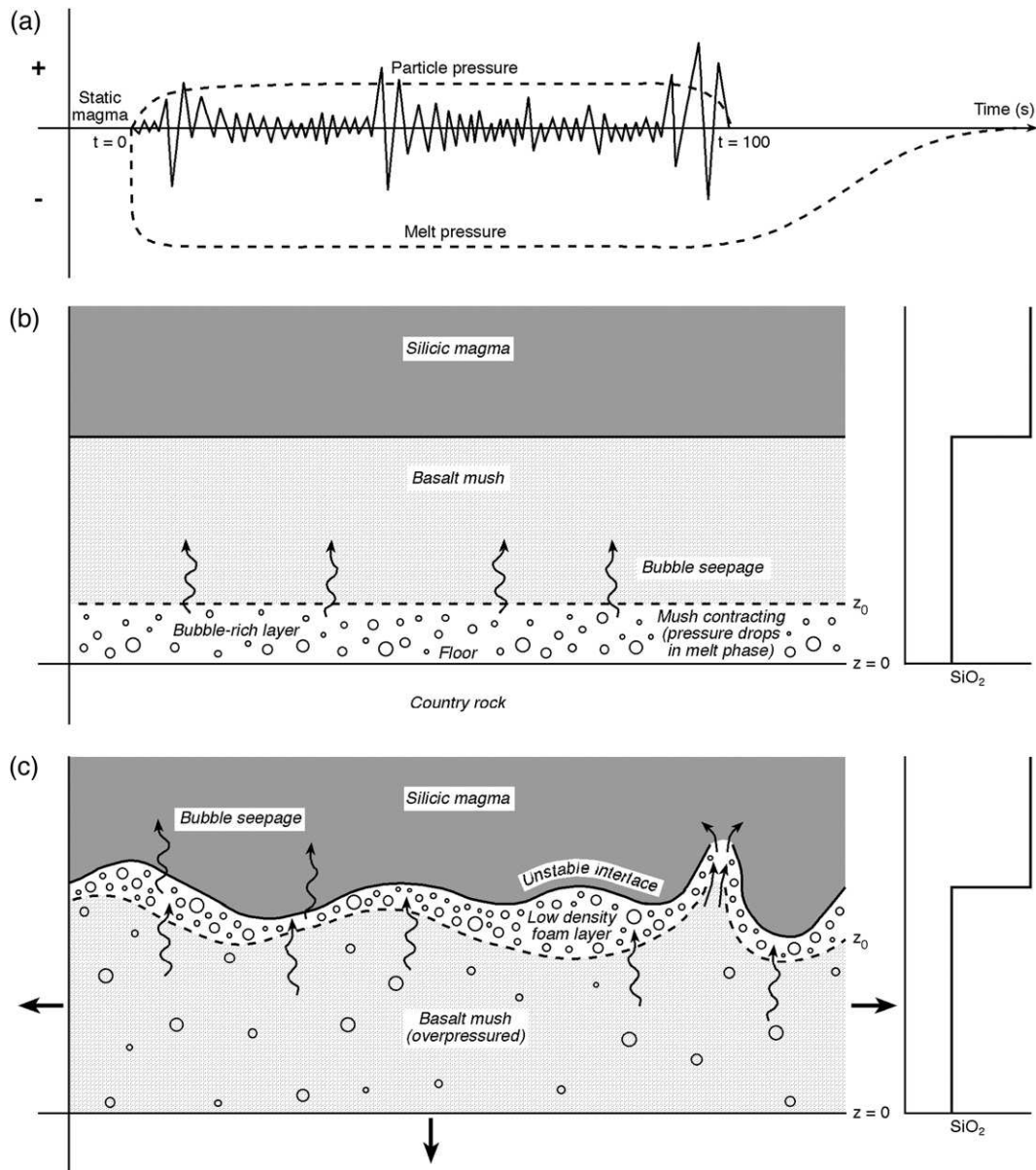


Fig. 4. Schematic diagram showing the proposed relationship between particle pressure and melt (fluid) pressure during seismically-induced magma excitation and instability development. (a). Curves summarising the complementary increase in particle pressure and simultaneous decrease in fluid (interstitial melt) pressure over the period of one earthquake cycle. Note that while the particle pressure 'switches off' as soon as the earthquake ends, the low pressure in the melt phase takes longer to decay back to its original (ambient) value. (b). Zoned and layered magma chamber geometry, showing a mafic layer close to the floor where bubble formation is predicted to occur while the earthquake is in progress. The presence of crystals should facilitate heterogeneous bubble formation. The solid phase (mush) undergoes contraction where the crystal content locally exceeds 50% (cf. Fig. 2). Once formed, bubbles are free to migrate upwards through the magma and collect at the basalt–silicic magma interface. (c). Conceptual development of the layered system some time after excitation. A low density foam layer has developed at the interface between the mafic and silicic magma, promoting local gravitational instability. Bubbles might rise into the overlying silicic layer if viscosity permits. The presence of new bubbles results in magma overpressure at the base of the chamber (outwards arrows).

consequences of our proposed effect. The agitation effect may also produce its own, possibly unique, internal seismic wave field – e.g. [Neuberg and O’Gorman \(2002\)](#) – that could provide a test of the model. The sequence of events following on from seismic wave interaction with magma are summarised in [Fig. 4](#).

5.1. Petrological implications

The agitation instability outlined here provides a mechanism for extracting a chemically evolved high melt fraction layer from a densely packed crystal mush on near-instantaneous time scales. In this way, seismically-induced fluidisation could act as a non-traditional differentiation mechanism that is independent of purely thermal effects ([Sparks, 2003](#)). The fact that our model requires both crystals and earthquakes as initial conditions, and results in fluid flow, nicely couples current geochemical work aimed at determining crystal residence times in magmas (e.g. [Wolff et al., 1999](#); [Hawkesworth et al., 2000](#); [Charlier et al., 2004](#)), with the dynamic aspects of the problem. Clearly a distinction has to be made between crystal residence times, which could be of the order of thousands of years ([Halliday et al., 1989](#); [Reid et al., 1997](#)), and eruption times, which are significantly less than this. While our model is consistent with longer term, secular processes in magma chambers (e.g. compaction) and gas sparging ([Sisson and Bacon, 1999](#)) involving buoyancy forces, the oscillation and pressure reduction instability provides a plausible mechanism for repeatedly separating crystals from contact with melt over the lifetime of the chamber, prior to eruption. Fluctuating episodes of crystal separation from liquid and allied degassing during the residence time of the crystal, up to the point of final withdrawal and eruption are predicted features of the model that may be testable geochemically, perhaps through detailed studies of melt inclusions and crystal zonation patterns. Indeed, should magma overpressure due to *in situ* bubble formation result in structural failure of the surrounding wall rocks, then full-scale eruption could result (see also [Phillips and Woods, 2001](#)). Another feature might be the rapid extrusion of mixed batches of broadly similar magma that show no petrological or geochemical evidence for significant mafic input, as in the Okataina and Taupo volcanic centres, New Zealand ([Smith et al., 2006](#)).

Finally, the theory as presented is used in a linear, small variation limit. Naturally when the amplitudes become very large ($\gg$ mean grain size), and as the viscosity of the interstitial melt phase increases, non-

linear corrections to the theory become very substantial (see [Melnik and Sparks, 2002](#) for the case of magma ascent), although the effect should also be present in coarser grained magmas where the excitation frequency is of comparable magnitude ([Gundogdu et al., 2003](#)).

6. Summary

We present a model of magma instability due to seismically-induced agitation. Three regimes are recognised as important in the development of the instability (static and secular) and are quantified analytically. The sequence of events is complex and interdependent, but results suggest that during excitation, the pressure in the solid phase (granular mush) at the floor of the chamber increases and at crystal contents in excess of 50%, net contraction of the magma layer takes place. Based on comparisons with experimental work, the corresponding fall in pressure in the melt phase ($\sim 1 \text{ MPa s}^{-1}$) is sufficient to form bubbles in viscous magma. A direct consequence of the proposed mechanism is rapid (near instantaneous) separation of melt from crystals. Bubbles formed during excitation may migrate upwards to the top of the magma layer (roof rock or interface with a different magma) and form layer capable of gravitational interaction (heat and mass transfer) with an overlying fluid. Finally, magma at the chamber base following the earthquake will be overpressured due to the presence of new bubbles. If the pressure exceeds the local strength of the surrounding country rocks, the chamber may fail resulting in eruption. None of this will happen in a liquid-dominated magma chamber subjected to seismic agitation. The notion that high levels of crystallinity ($>50\%$) in chambered magma constitute a rheological barrier reducing the risk of eruption may need revision.

Symbol	Meaning	Unit
A	Parameter defined in Eq. (22)	Pa m^{-1}
a	Mean particle radius	m
B	Parameter defined in Eq. (23)	Pa
C_E	$\partial \bar{E} / \partial \bar{\phi}$	Pa
C_R	$\partial \bar{R} / \partial \bar{\phi}$	Pa s m^{-2}
c	Proportionality constant for the particle pressure	–
E	Skeletal stiffness	Pa
$\mathbf{e}, \dot{\mathbf{e}}$	Strain tensor, strain rate tensor	–, s^{-1}
e	Void ratio $(1 - \bar{\phi}) / \bar{\phi}$	–
e_{zz}	Vertical strain	–
f_0	Grain material parameter	$\text{Pa}^{1/2}$
g	Acceleration due to gravity	m s^{-2}
H	Layer thickness	m
h	Gap width	m
p	Fluid pressure	Pa
$\bar{p}$	Particle pressure	Pa

p_s	Non-dimensional front factor for the particle pressure: Eq. (32)	–
p_f	Non-dimensional front factor for the particle pressure: Eq. (33)	–
q	Fluid discharge velocity $(1-\phi)u$	$m\ s^{-1}$
R	Fluid drag resistance	$Pa\ s\ m^{-2}$
r	Solid discharge velocity ϕv	$m\ s^{-1}$
t	Time	s
u	Fluid velocity	$m\ s^{-1}$
v	Solids velocity	$m\ s^{-1}$
x	Position vector	m
z	Vertical position	m
β	Compressibility	Pa^{-1}
γ	Damping coefficient	$Pa\ s$
δ	Kronecker delta	–
η	Melt viscosity	$Pa\ s$
λ	Inverse penetration depth (real part λ_0)	m^{-1}
π	Particle pressure	Pa
ϕ	Crystal volume fraction	–
χ	Attached fluid mass coefficient	–
ρ_f	Melt density	$kg\ m^{-3}$
ρ_s	Solids density	$kg\ m^{-3}$
$d\rho$	$\rho_s - \rho_f$	$kg\ m^{-3}$
Σ	Partial solid-phase stress	Pa
σ	Partial fluid-phase stress	Pa
τ	Intergranular stress	Pa
ω	Circular frequency	Hz

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