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GEOPHYSICS

A New Scheme for the Solution of Inverse Problems in Seismic Tomography

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Long-term experience in construction of seismic tomographic images in different regions of the world indicates the existence of the problem of ambiguity in results. For example, velocity sections obtained on the basis of numerical solutions of the systems of linear equations are not always comparable with the data of petrology and geology. The main cause is in the unknown level of the real error in each seismic observation, which leads to a distortion in the real image. A computational scheme in the linear formulation is suggested for the first time in the practice of the solution to tomographic problems, which allows us to avoid a significant influence of errors in the observations on the inverse result. The stability of the solution is guaranteed by a new regulation method in seismic tomography. The efficiency of the developed approach is proved by its ability to restore a complex model of the fracture zone within an admissible small error. Such a very high quality was not gained by the modern methods based on the standard approach.

The system of tomographic observations using a group of seismic stations usually located at the Earth's surface and a multitude of internal sources in the investigated volume of the medium is frequently overdetermined. This means that the number of seismic rays is significantly greater than the number of comparatively small blocks of the medium, for which the characteristics of seismic waves are restored. Reliable reconstruction of tomographic images requires that the information contained in time discrepancies of the transit times of rays should be independent. Information is usually repeated for closely located rays formed by the clusters of seismic activity and common recording stations. Therefore, we propose to start data inversion by preliminary decimation of the number of such rays. The most interesting situation is when a beam of rays of such paths has one or more values of discrepancies from the mean value obtained for close values. Do we have to trust this discrepancy or exclude it? The discrepancy can be related to several causes: (a) an error in determining the source coordinates; (b) vague (noisy) arrival of the seismic wave from the given source to the specific station; (c) poor seismogram record or inadequate recording conditions; and (d) arrival of the seismic wave generated by a noisy object (fracture zone, iron orebodies, oil pools, and others). We suggest a probabilistic differential approach to separate the error effect in data (a)–(c) from the useful information about anomalous zones (d).

Let us consider a system of linear equations, which is the basis of the modern seismic tomography

$$Az = u + \varepsilon, \tag{1}$$

where z is an unknown vector, whose components are specific parameters that characterize the blocks of an inhomogeneous medium; u is the vector known from observations (in our case, the vector of discrepancy of transit times); A is a rectangular matrix, whose elements correspond to the segments of seismic rays in the blocks of the medium; and ε is a vector of observation errors. The number of lines m in matrix A is determined by the number of seismic rays, while the number of columns or blocks n is specified by the selected parameterization of the medium. The goal of the seismic tomography is to use all available information maximally. Therefore, in practice $m \gg n$. The standard least square method (LSQ) operates with a square symmetric matrix with dimension $n \times n$. In order to use this method, both parts of Eq. (1) are additionally multiplied by the matrix transposed to the initial matrix A [1]. The obtained system has a unique solution if it is not degenerated. In the opposite case, we can speak about an approximate solution that satisfies the minimum of the LSQ functional. In seismic tomography, the scaled norm of vector z is minimized together with the LSQ functional to obtain stable solutions with respect to the errors of observations [2–4].

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Unfortunately, it is not always possible to estimate correctly the reliability of the tomographic solution [5]. The initial matrix is large and very sparse. Researchers apply various algorithms to calculate the resolution matrix, which could characterize the accuracy of the solution. However, the main cause of the problem lies in the existence of large errors in the observation vector u and in the accumulation of computational errors in recurrence relations for large and sparse data sets after a large number of iterations. Therefore, we suggest reducing the problem of large dimension to a multitude of problems of small dimension, which could be solved with greater accuracy to obtain correct estimates of the quality. The final solution is chosen on the basis of the most probable solutions of small subproblems. Thus, the suggested probabilistic differential approach tends to select the part of the initial system that maximally suits the correctness of the problem formulation.

The lines of initial matrix A have different lengths depending on the number of blocks of the medium passed by the seismic rays. The columns of the matrix correspond to a specific block and have many zeros because not all rays but only a part of them crosses the block. Let us separate smaller matrices from the initial matrix. Each of them would be characterized by a data completeness feature; i.e., the seismic rays of the corresponding subsystem pass through the same blocks of the medium. Physically, this means the consideration of certain arrivals of waves propagating from the clusters of seismic activity at different depths and recorded by an individual seismic station. A separate subsystem can be either overdetermined or underdetermined. In the first case, different right-hand parts can correspond to two similar lines of the matrix due to the observation errors (the information is excessive). In the other case, the equations can repeat each other and the number of independent equations would be insufficient with respect to the number of unknown parameters. Let us consider here the solution of overdetermined problems.

The standard LSQ gives us the possibility to multiply both parts of the subsystem by a transposed submatrix to obtain a convenient system for computation, but this system has a smaller size and symmetric positively definite square matrix. We note that all equations of the subsystem in this case would contribute to the determined components of vector z. However, each discrepancy can either reflect anomalous deviations from the specified model or can be a result of errors in the observation data. We suggest another (relative to the LSQ) algorithm of the solution of an overdetermined system to control the influence of an unpredicted combination of errors on the results of inversion.

Let us select those equations from the subsystem that agree with the condition of the uniqueness of numerical solution of any system known from linear algebra. Namely, the rank of the selected matrix should be equal to the rank of the corresponding larger matrix containing the column of observed data, which has the number of lines equal to the number of unknown parameters. In other words, we select n_S independent values from m_S initial equations and form the basic system. Let us address the elementary probability theory to determine the consistency of this system.

Let us consider two random values: ξ is a column vector of basic system observations with dimension $n_S \times n_S$ and η is an arbitrary column vector of the basis matrix. According to [6], the greater the correlation coefficient $\rho(\xi, \eta)$ between ξ and η, the smaller the root-mean-square error of estimate Δ:

$$\Delta = D\eta(1 - \rho^2(\xi, \eta)), \tag{2}$$

where $D\eta$ is the dispersion of η. It is clear that if $\rho(\xi, \eta)$ is equal to 1, Δ is equal to 0.

Hence, if all calculated correlation coefficients of the vector of observations with the columns of the basis matrix are equal to 1, the system is consistent. The inverse matrix for the obtained system and components of vector z can be calculated easily.

It is clear that the basic solution is one of the possible solutions of an overdetermined subsystem with dimension $m_S \times n_S$. Test calculations demonstrated that if the data of m_S measurements do not contain errors, the basic solution coincides with the LSQ solution. However, if observation errors exist, these solutions are different. The basic solutions satisfy selected basic equations, while the LSQ corresponds to all equations of the overdetermined subsystem. Where is the guarantee that basic solutions are related to the properties of the inhomogeneous medium rather than the chosen erroneous observations? In order to find a stable solution, we regulate the process by constructing a sequence of components for basic solutions found for the set of filled subsystems related to the same system block. As was mentioned before, each filled subsystem is related to an individual seismic station. The existence of approximately equal values in the constructed sequence indicates that the basic solutions stably characterize the system from different azimuthal directions, each of which is specified by its seismic station. Approximately equal values can be defined as a converging subsequence. Thus, if we could find such a sequence, it will be a regularized sequence according to the theory of regularization [7].

We note that the error of each of the basic solutions found should be calculated with respect to each of m_S equations in order to take into account all available information about the medium. The traditional LSQ is useful here, and the error can be estimated by standard methods. However, in this case, we avoid the problem of computational rounding because we operate with nonthinned arrays of small dimension. If it is possible to determine approximately equal basic solutions for the considered block of the medium characterized by vector component z, it is reasonable to select the solution with a minimal LSQ error.

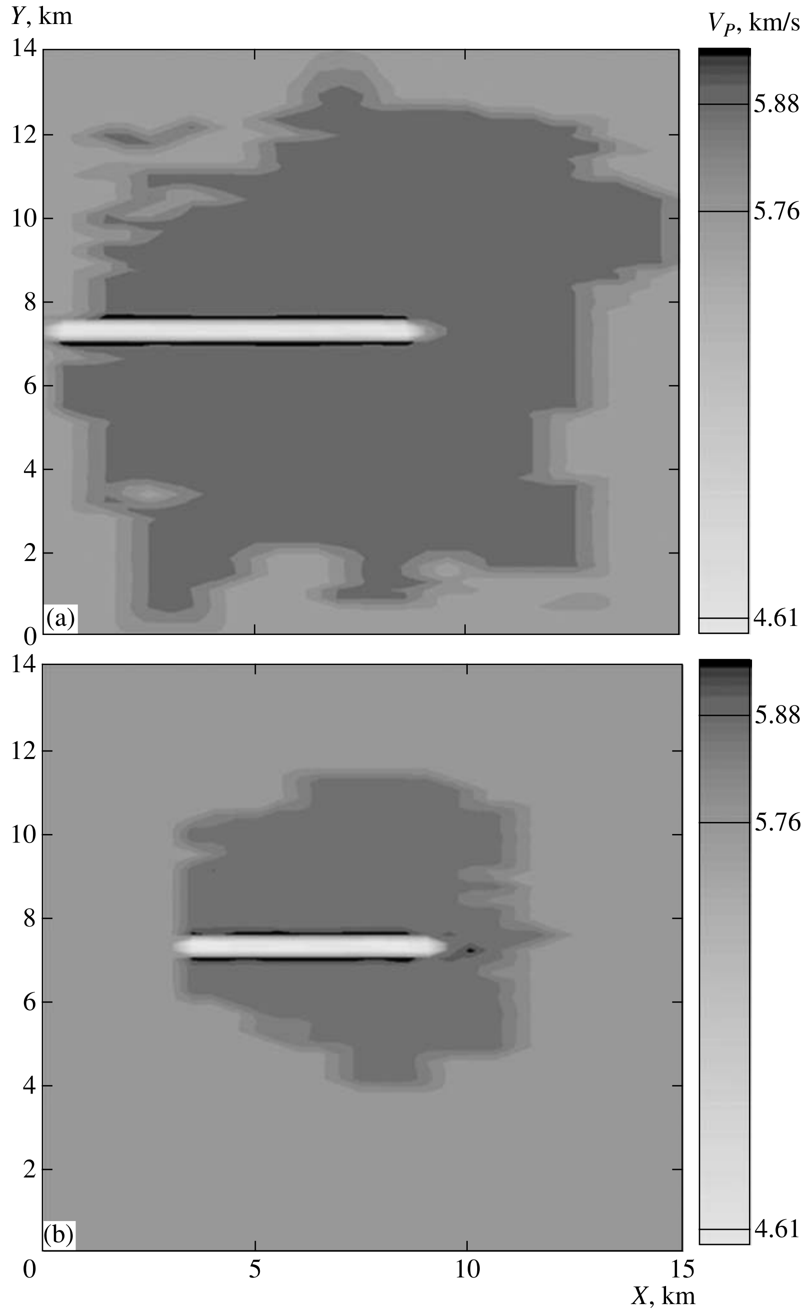

Fig. 1. *XOY* projection of the synthetic velocity model of a fracture zone in the depth range *Z* from 0.6 to 0.9 km. The projection of the inclined plane of the fracture is denoted by a white band. The high-velocity zone surrounding the fracture is colored dark gray. It is shown over the background of mean velocity 5.6 km/s colored gray. (a) Real image (includes regions of a medium crossed by at least one seismic ray); (b) image restored by the suggested probabilistic differential approach (corresponds to the region of the medium most densely crossed by seismic rays).

It is possible to estimate the set of components of unknown vector z by applying the approach described above. Substituting the found values into Eq. (1) and subtracting them from the observations in the right-hand part, we continue the iteration process of determining the further components by investigating the filled subsystems, whose dimension decreases due to the formulation of the problem. It is recommended to take into account the specific features of seismic tomography problems and take the first steps for the subsystems with small values of n_S, which correspond to surface layers. In this case, seismic rays do not contain serious errors of linearization, which are increased with consideration of deeper complex media. While passing to the components of vector z, which characterize deeper structures, it is necessary to regulate the val-

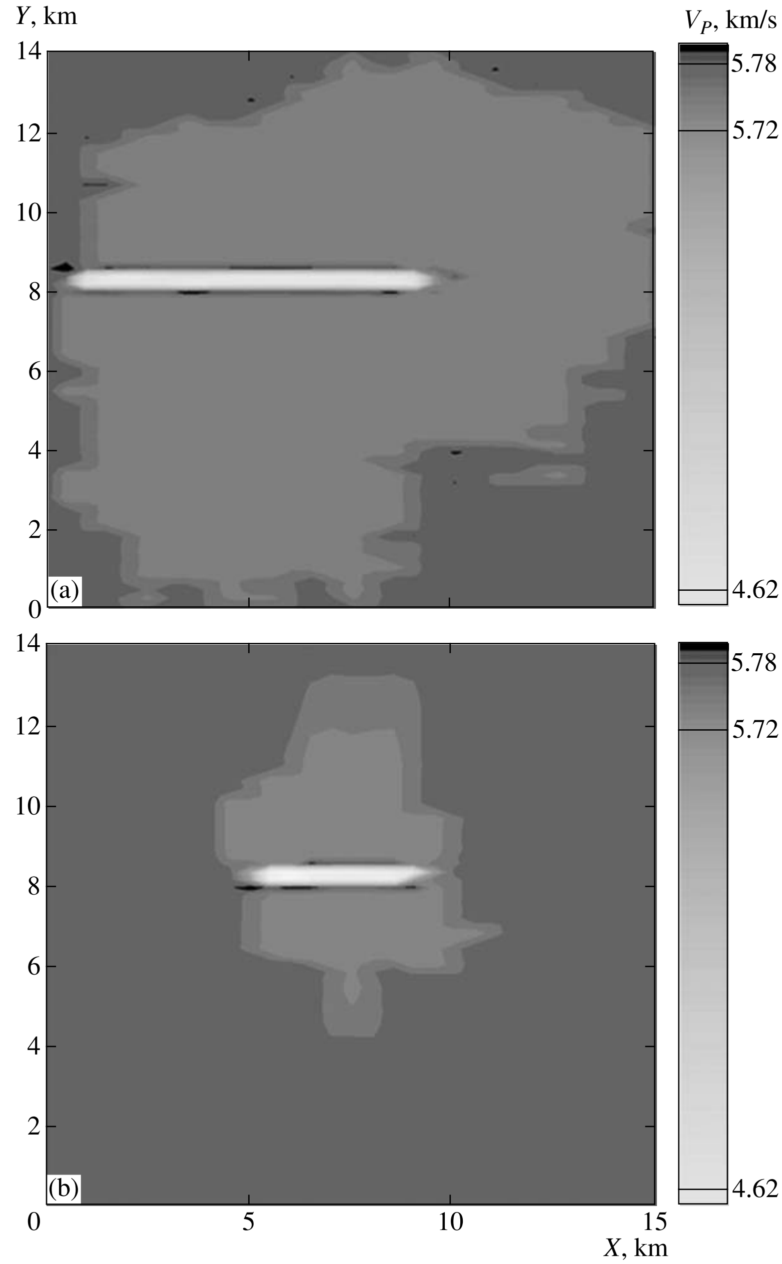

Fig. 2. *XOY* projection of synthetic velocity model of a fracture zone in the depth range *Z* from 3.9 to 4.2 km. The projection of inclined plane of the fracture is denoted by a white band. Low-velocity zone (light gray) surrounding the fracture is shown over the background of mean velocity 5.78 km/s (dark gray). (a) Real image (includes regions of a medium crossed by at least one seismic ray); (b) image restored by the suggested probabilistic differential approach (corresponds to the region of a medium most densely crossed by seismic rays).

ues of the LSQ error accumulated with the iterations and select only the basic solutions within the admissible LSQ accuracy.

The suggested probabilistic differential approach was tested using numerous synthetic and real data. Figures 1 and 2 illustrate the results of tomographic reconstruction of a complex synthetic model of a fracture zone formed after an earthquake with magnitude 6.8 on September 14, 1984, in the western Nagano region (central Japan). The data of more than 8000 aftershock earthquakes recorded by 30 stations were cordially given to us by the Laboratory of the National Research Institute for Earth Science and Disaster Prevention (Japan) headed by Prof. Horiuchi, whose ideas stimulated the development of the new inversion approach.

The real model is characterized by low velocity (4.6 km/s) within the fracture plane (0.3 km thick), which is surrounded by a high-velocity (5.87–5.9 km) zone down to a depth of 2.1 km, and a relatively low-velocity (5.7–5.73 km) zone at deeper levels. Projections of the fracture plane are shown with a white line, which is surrounded in the depth range of 0.6–0.9 km by a high-velocity zone of dark gray color (Fig. 1a) and in the depth range of 3.9–4.2 km by a low-velocity zone of light gray color (Fig. 2a). The overdetermined system with dimension 103 148 × 19 399 and uniformly distributed systematic errors of 0.01 s was solved using the new method. The specified model was restored practically without errors in the region of dense ray coverage. The maximal difference between the real and determined values is equal to 5% (Figs. 1b, 2b). Such high quality of the solution confirmed the correctness of the suggested scheme for the solution of the inverse problem, which can be applied for work with field seismic observations.

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