

Modified imaging principle: An alternative to preserved-amplitude least-squares wave-equation migration

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ABSTRACT

Impedance contrast images can result from a least-squares migration or from a modified imaging principle. Theoretically, the two approaches should give similar results, but in practice they lead to different estimates of the impedance contrasts because of limited acquisition geometry, difficulty in computing exact weights for least-squares migration, and small contrast approximation. To analyze those differences, we compare the two approaches based on 2D synthetics. Forward modeling is either a finite-difference solver of the full acoustic wave equation or a one-way wave-equation solver that correctly models the amplitudes. The modified imaging principle provides better amplitude estimates of the impedance contrasts and does not suffer from the artifacts attributable to diving waves, which can be seen in two-way, least-squares migrated sections. However, because of the shot-based formulation, artifacts appear in the modified imaging principle results in shadow zones where energy is defocused. Those artifacts do not exist with the least-squares migration algorithm because all shots are processed simultaneously.

INTRODUCTION

Preserved-amplitude migration has been and still remains a challenge in seismic prospecting. With ray-based modeling and high-frequency assumptions, true-amplitude migration has been studied extensively during the last three decades (see, for instance, Bleistein et al., 2001, or Schleicher et al., 2007, and the references therein). Because of the limitations of the ray-based algorithm when multipathing occurs, such as around salt domes with rough interfaces, the geophysical community uses wave-equation migration. In this pa-

per, we only address finite-frequency wave-equation migration.

Wave-equation migration formulas generally are based on the classic imaging principle (Claerbout, 1971) or on a least-squares formulation (Tarantola, 1984). These two approaches do not estimate the same physical quantities. The classic imaging principle approximates the (shot/angle-dependent) reflection coefficients; least-squares migration generally estimates elastic-parameter perturbations/contrasts. The formulation through Claerbout's imaging principle allows us to carry out amplitude variation with angle (AVA) analysis. However, because of limited receiver coverage, artifacts appear in a one-shot image (obtained from a single shot). With multi-shot data sets and ray-based modeling, this problem can be overcome by working with offset gathers. This is no longer possible with finite-difference modeling. With a one-way scheme, Claerbout (1985) proposes the double square root formulation. Following this approach and the work of Sava and Fomel (2003), Joncour et al. (2005) develop a preserved-amplitude wave-equation migration method.

To obtain the elastic-parameter perturbations without conducting AVA analysis, we cannot apply the imaging principle directly because of its angle dependency. However, the least-squares formulation provides this information immediately. Moreover, the least-squares formulation processes the full multishot data set at once, which reduces the artifacts resulting from energy defocusing (in shadow zones). We can formulate least-squares migration with a survey-sinking approach (Kuhl and Sacchi, 2003) or a shot-profile approach (Rickett, 2003). In this paper, we consider the latter.

The main difficulty of least-squares migration is computing optimal migration weights if one wants to avoid an iterative scheme that minimizes a least-squares misfit function. The least-squares migration weights are computed from an approximation of the pseudoinverse of the Hessian (the second derivative of the least-squares misfit). In practice, we approximate this pseudoinverse only by a diagonal matrix because we cannot afford to compute the full Hessian. This limits the validity of least-squares migration because the Hes-

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sian is not diagonally dominant at finite frequencies (Plessix and Mulder, 2004).

Shin et al. (2001) and Plessix and Mulder (2004) discuss least-squares migration and propose new approximations of the Hessian of the least-squares misfit to construct preserved-amplitude least-squares migrations. Their results are based on two-way (full) wave-equation modeling; however, we can also apply them with one-way wave-equation modeling. Guitton (2004) proposes approximating the inverse of the Hessian with filters determined from the cascade of migration and demigration operations. The choice of filters is not unique in this approach and is not considered in our current article.

Kiyashchenko et al. (2007) develop a modification of the imaging principle that gives slowness perturbations instead of angle-dependent reflection coefficients. The modification consists of multiplying the crosscorrelation between incident and back-propagated fields by the squared cosine of the reflection angle. Although keeping the common-shot formulation, this modification reduces the shot dependency of the shot image and thereby estimates slowness perturbations after stacking the shot images over shots. This shot-based modified imaging principle is similar to the ray-based inversion formula of Bleistein et al. (2001) but works with any type of modeling, especially finite-difference wave-equation modeling (Kiyashchenko et al., 2007). Thus, it is an alternative to least-squares migration and results in true-amplitude slowness perturbation maps, a quantitative structural image of the slowness contrasts. With continuous and large acquisition coverage, it supersedes one-step least-squares migration algorithms, which rely on a diagonal approximation of the Hessian. Unfortunately, this assumption is often violated in practice, and artifacts may appear in the presence of strong energy defocusing.

In this article, we compare the stack-migrated images of slowness perturbations obtained with least-squares migration (with diagonal approximation of the Hessian) and the modified imaging principle on 2D synthetic data sets using finite-difference wave-equation schemes. We consider a two-way wave-equation scheme and a multi-one-way (Kiyashchenko et al. 2005a) wave-equation scheme. The thin-slab multi-one-way scheme correctly approximates the amplitudes of the downgoing waves.

The article is organized as follows. First, we briefly describe the least-squares migration formula and the modified imaging principle. Then we present the thin-slab multi-one-way method. Next, we show two synthetic examples. Finally, we draw some conclusions.

MODIFIED IMAGING PRINCIPLE AND LEAST-SQUARES MIGRATION

Assuming an acoustic earth with constant density, least-squares migration estimates slowness perturbations. We obtain a similar estimate with a modified shot-based imaging principle. This latter assumes a regular, dense acquisition geometry, whereas the least-squares formulation can handle irregular acquisition geometries.

Modified imaging principle

A shot migration with the classic imaging principle (Claerbout, 1971) approximates the reflection coefficients that depend on incident angles or shot positions. With multishot data, the stack of the common-shot images obtained from migrating the shot gathers with the classic imaging principle produces a structural image of the earth. However, the amplitudes are distorted because of the shot de-

pendency. To handle multishot data, Claerbout (1985) proposes using a double square root formalism.

With a different point of view and to retrieve a good approximation of the amplitudes of slowness perturbations while keeping a shot-based formula, Kiyashchenko et al. (2005b) proposes a modification of the classic imaging principle. At small angles of incidence, the reflection coefficient is equal to the slowness contrast divided by the squared cosine of the reflection angle (Bleistein et al., 2001). Therefore, if we multiply the integrand of the imaging principle, which corresponds to the crosscorrelation between the incident and the back-propagated fields, by the squared cosine of the reflection angle, we obtain an estimate of the slowness contrast instead of the reflection coefficient.

We implement this idea in the context of wave-equation migration by computing the squared cosine of the reflection coefficient from the incident and back-propagated fields. This leads to the following modified imaging principle I_M (see Appendix A):

$$I_M(\mathbf{x}_s, \mathbf{x}) = \frac{\int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{|\omega i|}}{\pi A(\mathbf{x}_s, \mathbf{x})} \int_{\omega_{\min}}^{\omega_{\max}} d\omega \operatorname{sgn}(\omega) \left(u_d(\mathbf{x}_s, \mathbf{x}, \omega) \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega) - \frac{1}{k^2} \left[\frac{\partial u_d(\mathbf{x}_s, \mathbf{x}, \omega)}{\partial x} \frac{\partial \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega)}{\partial x} - \Lambda_a u_d(\mathbf{x}_s, \mathbf{x}, \omega) \Lambda_a \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega) \right] \right),$$

$$A = \int_{\omega_{\min}}^{\omega_{\max}} d\omega u_d \bar{u}_b. \quad (1)$$

Here, $u_d(\mathbf{x}_s, \mathbf{x}, \omega)$ is the incident wavefield generated by a source at $\mathbf{x}_s$, $\mathbf{x}_r$ is the receivers, ω is angular frequencies, and $u_b(\mathbf{x}_s, \mathbf{x}, \omega)$ is the back-propagated wavefield. The overbar denotes the complex conjugate, and $\omega_{\min}$ and $\omega_{\max}$ are the minimum and maximum angular frequencies. If the integration is done only over the positive frequencies, the real part of the right-hand side of equation 1 should be taken. In the imaging principle, the data are interpreted as a measure of the wavefield on a boundary. This means they should correspond to a boundary condition in the modeling schemes.

In theory, this approach is valid only for purely acoustic data. We often make a similar assumption with least-squares migration. This limits the application range of the method to medium-offset data because, at long offsets, the amplitudes depend significantly on shear-velocity contrasts. Equation 1 gives a band-limited approximation of the slowness contrast, i.e., the slowness contrast convolved with a spatial filter depending on acquisition coverage and the frequency range of the data. Under the high-frequency assumption, this formula is similar to the imaging formula of Bleistein et al., 2001 (Kiyashchenko et al., 2007).

Equation 1 is a common-shot formula. With a multishot data set, the final slowness contrast image is obtained by stacking all common-shot images and dividing the results by the number of shots, assuming infinite receiver coverage. In practice, finite coverage can cause significant errors in amplitude estimates (see Appendix B). We reduce these errors by stacking the shot-based slowness perturbation image $\sum_{\mathbf{x}_s} I_M(\mathbf{x}_s, \mathbf{x})$ with the receiver-based slowness perturbation im-

age $\sum_r I_M(\mathbf{x}_r, \mathbf{x})$. We compute $I_M(\mathbf{x}_r, \mathbf{x})$ after resorting the data by receivers using reciprocity between sources and receivers. We then obtain the approximate slowness contrast from a multishot data set by

$$I_{ST}(\mathbf{x}) = \frac{1}{2} \left(\sum_{\mathbf{x}_s} I_M(\mathbf{x}_s, \mathbf{x}) + \sum_{\mathbf{x}_r} I_M(\mathbf{x}_r, \mathbf{x}) \right), \quad (2)$$

where ST stands for stack. This approach obviously is twice as expensive as the conventional wave-equation migration procedure. However, it improves the amplitude estimates of the final stack and removes most of the coverage effects (Kiyashchenko et al., 2005b).

Because of its shot-based formulation, expression 1 suffers from artifacts in the zones of defocusing energy when $A(\mathbf{x}_s, \mathbf{x})$ becomes too small. We avoid the division by a too small quantity by replacing $A(\mathbf{x}_s, \mathbf{x})$ by $A(\mathbf{x}_s, \mathbf{x}) + \varepsilon$, with ε being a regularization parameter.

Least-squares approach

We can also estimate slowness perturbations by minimizing the least-squares misfit between data and computed synthetics. In this approach, the data are no longer interpreted as measurements on the boundary of the domain but as point measurements. Therefore, any type of acquisition, including sparse acquisitions, can be handled.

The least-squares misfit reads

$$J(r) = \frac{1}{2} \sum_{\mathbf{x}_s} \sum_{\mathbf{x}_r} \sum_{\omega} |c(\mathbf{x}_s, \mathbf{x}_r, \omega; r) - d(\mathbf{x}_s, \mathbf{x}_r, \omega)|^2, \quad (3)$$

where $d(\mathbf{x}_s, \mathbf{x}_r, \omega)$ are the data and $c(\mathbf{x}_s, \mathbf{x}_r, \omega; r)$ the synthetics, depending on the slowness contrast r . The slowness is equal to $\sigma_0(1 + r)$, where σ_0 is the background slowness used in the migration algorithms. The summation is carried out over $\mathbf{x}_s$, $\mathbf{x}_r$, and ω . We assume a linearized model, so the optimal slowness contrast $\hat{r}$ satisfies

$$H\hat{r} = -g, \quad (4)$$

where g is the gradient of J evaluated at $r = 0$ and H is the Hessian.

With $G(\mathbf{x}_s, \mathbf{x}_0, \omega)$ the Green's function valued at $\mathbf{x}$ because of a Dirac function at $\mathbf{x}_s$, the gradient and the Hessian read (Plessix and Mulder, 2004)

$$\begin{aligned} g(\mathbf{x}) &= 2\sigma_0^2(\mathbf{x}) \operatorname{Re} \left(\sum_{\omega} \omega^2 \sum_{\mathbf{x}_s} f_s(\omega) G(\mathbf{x}_s, \mathbf{x}, \omega) \right. \\ &\quad \times \sum_{\mathbf{x}_r} G(\mathbf{x}, \mathbf{x}_r, \omega) [\bar{c}(\mathbf{x}_s, \mathbf{x}_r, \omega; 0) - \bar{d}(\mathbf{x}_s, \mathbf{x}_r, \omega)] \Big), \\ H(\mathbf{x}, \mathbf{y}) &= 4\sigma_0^2(\mathbf{x}) \sigma_0^2(\mathbf{y}) \operatorname{Re} \left(\sum_{\omega} \omega^4 \sum_{\mathbf{x}_s} |f_s(\omega)|^2 \right. \\ &\quad \times G(\mathbf{x}_s, \mathbf{x}, \omega) \bar{G}(\mathbf{x}_s, \mathbf{y}, \omega) \sum_{\mathbf{x}_r} G(\mathbf{x}, \mathbf{x}_r, \omega) \\ &\quad \times \bar{G}(\mathbf{y}, \mathbf{x}_r, \omega) \Big), \end{aligned} \quad (5)$$

where f_s is the source signature. The factors $2\sigma_0^2(\mathbf{x})$ and $4\sigma_0^2(\mathbf{x})\sigma_0^2(\mathbf{y})$ appear because we define r by $(\sigma - \sigma_0)/\sigma_0$, but Plessix and Mulder (2004) use $\sigma^2 - \sigma_0^2$. The incident wavefield is $u(\mathbf{x}_s, \mathbf{x}, \omega) = f_s(\omega)G(\mathbf{x}_s, \mathbf{x}, \omega)$. The Born approximation at $r = 0$ gives $c(\mathbf{x}_s, \mathbf{x}_r, \omega; r) = 0$, and $\sum_{\mathbf{x}_r} G(\mathbf{x}, \mathbf{x}_r, \omega) \bar{d}(\mathbf{x}_s, \mathbf{x}_r, \omega)$ is the back-propagated wavefield. The gradient is kinematically equivalent to the stack of the migrated shot sections (Lailly, 1983). We obtain a least-squares migration formula when $\hat{r}$ is approximated in one step. Itera-

tive optimization of equation 4 leads to iterative least-squares migration.

The Hessian is too large to be computed for a practical case. We obtain an affordable method with a diagonal approximation of the inverse of the Hessian. The least-squares migration then reads

$$I_{LS}(\mathbf{x}) = -\frac{1}{\sigma_0^4(\mathbf{x})} K(\mathbf{x}) g(\mathbf{x}), \quad (6)$$

with $(1/\sigma_0^4 K)$ the diagonal approximation.

We can find several approximations in the literature. Shin et al. (2001) propose an approximation of the diagonal of the inverse of the Hessian (type 1 preconditioner) (Plessix and Mulder, 2004):

$$K^{(1)}(\mathbf{x}) = \left(\sum_{\omega} \omega^4 \sum_{\mathbf{x}_s} |u(\mathbf{x}_s, \mathbf{x}, \omega)|^2 \right)^{-1}. \quad (7)$$

Plessix and Mulder (2004) take into account the limited coverage and nondiagonal structure of the Hessian and propose this approximation (type 2 preconditioner):

$$\begin{aligned} K^{(2)}(\mathbf{x}) &= \left(\sqrt{\frac{z}{\sigma_0(\mathbf{x})}} \sum_{\omega} \omega^4 \sum_{\mathbf{x}_s} |u(\mathbf{x}_s, \mathbf{x}, \omega)|^2 \right. \\ &\quad \times \left[\operatorname{asinh} \left(\frac{x_r^{\max}(\mathbf{x}_s) - x}{z} \right) \right. \\ &\quad \left. \left. - \operatorname{asinh} \left(\frac{x_r^{\min}(\mathbf{x}_s) - x}{z} \right) \right] \right)^{-1}, \end{aligned} \quad (8)$$

where $x_r^{\max}$ and $x_r^{\min}$ are the maximal and minimal receiver x -coordinates of the shot $\mathbf{x}_s$ (all sources and receivers are assumed to be at the same depth). The nondiagonal structure of the Hessian is taken into account with a mass-lumping approach. This explains the term $\sqrt{z/\sigma_0(\mathbf{x})}$. It is impossible to compute off-diagonal terms efficiently, so we assume that the sum of the nondiagonal terms is proportional to the Fresnel zone at the subsurface point $\mathbf{x}$. A third approximation (type 3 preconditioner) is

$$\begin{aligned} K^{(3)}(\mathbf{x}) &= \left(\frac{1}{\sigma_0^2(\mathbf{x})} \sum_{\omega} \omega^4 \sum_{\mathbf{x}_s} |u(\mathbf{x}_s, \mathbf{x}, \omega)|^2 \right. \\ &\quad \times \left[\operatorname{asinh} \left(\frac{x_r^{\max}(\mathbf{x}_s) - x}{z} \right) \right. \\ &\quad \left. \left. - \operatorname{asinh} \left(\frac{x_r^{\min}(\mathbf{x}_s) - x}{z} \right) \right] \right)^{-1}. \end{aligned} \quad (9)$$

It differs from the second one only by the mass-lumping factor. Here, $1/\sigma_0^2(\mathbf{x})$ replaces $\sqrt{z/\sigma_0(\mathbf{x})}$.

THIN-SLAB MULTI-ONE-WAY MODELING

Correctly computing wavefield amplitudes is the first step toward true-amplitude migration. Classic one-way schemes generally do not provide reliable amplitudes (Zhang et al., 2003). Here, we consider the multi-one-way approach of Kiyashchenko et al. (2005a) to improve amplitude estimates.

The scheme is based on the following factorized two-way wave equation:

$$\left(\frac{\partial}{\partial z} + i\Lambda_a\right)\left(\frac{\partial}{\partial z} - i\Lambda_a\right)u = f + Eu. \quad (10)$$

Here E is the error term

$$Eu = \left((\Lambda_a^2 - \Lambda^2) - i\frac{\partial \Lambda_a}{\partial z}\right)u, \quad (11)$$

f is the source term, u is the pressure field, and Λ is the square root operator $\sqrt{k^2(x, z) + \Delta_x}$, with k the wavenumber and $\Delta_x = \partial^2/\partial x^2$ in 2D space and $\Delta_x = \partial^2/\partial x^2 + \partial^2/\partial y^2$ in 3D space. The value Λ_a is a local approximation of Λ , e.g., the screen propagator (Wu, 1994; Wu and Huang, 1995; Jin et al., 1998; Xie and Wu, 1996, 1998; Jin, 1999) or the rational Padé approximation (Ristow and Ruhl, 1997; Collino and Joly, 1999).

In the multi-one-way method, we approximately solve equation 10 with the perturbation theory, assuming small slowness contrasts. This differs from the approach proposed by Zhang et al. (2003). The

zero-order approximation u_0 of the solution satisfies the downgoing one-way wave equation (Kiyashchenko et al., 2005a):

$$\left(\frac{\partial}{\partial z} + i\Lambda_a\right)\left(\frac{\partial}{\partial z} - i\Lambda_a\right)u_0 = f. \quad (12)$$

The first-order approximation u_1 satisfies

$$\left(\frac{\partial}{\partial z} + i\Lambda_a\right)\left(\frac{\partial}{\partial z} - i\Lambda_a\right)u_1 = Eu_0, \quad (13)$$

which is the product of an upgoing operator with a downgoing operator and $u \approx u_0 + u_1$.

Kiyashchenko et al. (2005a) describe the global approach, in which each one-way scheme is solved sequentially in the entire domain — from top to bottom for the downgoing scheme and from bottom to top for the upgoing scheme. This global scheme consists of solving three one-way wave equations with nonzero right-hand sides. The approach models the primary reflected waves. The presence of the primary reflected waves and diving waves in the incident and back-propagated fields may create artifacts in the migrated image (Mulder and Plessix, 2004a; Yoon et al., 2004). We avoid those artifacts if we only consider the downgoing wavefield. Our thin-slab multi-one-way scheme improves the amplitude estimates of the downgoing wavefield by modeling the transmission effects. This approach does not account for the reflection effects.

The thin-slab multi-one-way scheme to compute $u \approx u_0 + u_1$ from depth z to $z + \Delta z$ (Δz is the depth increment) consists of three steps:

- 1) One step down. Solve from z to $z + \Delta z$:

$$\frac{\partial u_0}{\partial z} = i\Lambda_a u_0, \quad (14)$$

with $u_0(z) = u(z)$. At the source depth, $z = 0$ and $u(0) = (1/2i)\Lambda_a^{-1}f$.

- 2) One step up. Solve from $z + \Delta z$ to z :

$$\left(\frac{\partial}{\partial z} + i\Lambda_a\right)u_{1/2} = Eu_0, \quad (15)$$

with $u_{1/2}(z + \Delta z) = 0$.

- 3) One step down. Solve from z to $z + \Delta z$:

$$\left(\frac{\partial}{\partial z} - i\Lambda_a\right)u_1 = u_{1/2}, \quad (16)$$

with $u_1(z) = -(1/2i)\Lambda_a^{-1}u_{1/2}(z)$.

A discussion on the choice of the boundary conditions can be found in Kiyashchenko et al. (2005a).

We now illustrate the improvement brought by the thin-slab multi-one-way scheme by computing the downgoing wavefield with the classic one-way approach, which corresponds to step 1, and with the thin-slab multi-one-way approach. The computation is carried out at 14 Hz on a 2D smoothed section of the SEG/EAGE model (Figure 1). We compare those wavefields with the one obtained from a two-way frequency-domain (FD) finite-difference scheme.

The absolute relative errors are plotted in Figure 2. The source is situated at the top of the model at $x = 3$ km. In a cone originating at the source point, the relative errors are reduced by about one order of magnitude to just a few percent with the thin-slab multi-one-way

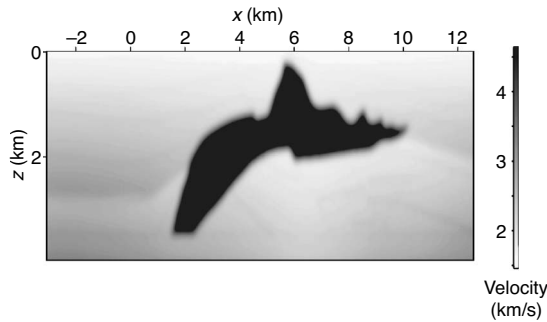


Figure 1. A smoothed section of the acoustic velocity of the SEG/EAGE salt model in kilometers/second.

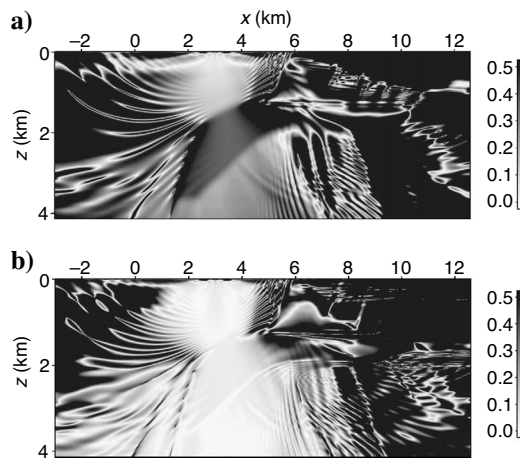


Figure 2. Absolute relative errors between (a) the one-way wavefield computed with the classic one-way scheme and the two-way wavefield and (b) the one-way wavefield computed with the thin-slab multi-one-way scheme and the two-way wavefield. The velocity model is the smoothed section of the SEG/EAGE salt model (Figure 1). The white zone corresponds to a relative error of 0% and the black zone corresponds to a relative error larger than 50%.

scheme (Figure 2b) compared to the classic one-way scheme (Figure 2a). The thin-slab multi-one-way scheme correctly estimates the amplitudes of the downgoing wave in the cone of validity of the one-way approximation. However, it is three to five times more expensive than the classic one-way scheme.

We have not really compared the computational cost of this approach with the two-way solver. In two dimensions and with a large number of shots, it is probably as expensive as the two-way solver (see Mulder and Plessix [2004a] for a comparison of one-way and two-way solvers' computational cost in two dimensions). A 3D one-way approach is one or two orders of magnitude less expensive than a two-way solver, depending on the implementation. In the frequency domain, it is possible to reduce computation costs by adopting grid spacing according to frequency and by reducing the number of frequencies used for migration (Mulder and Plessix, 2004b). Therefore, a 3D multi-one-way approach should be at least several times faster than a two-way approach.

Differences in implementation

Whereas the two-way scheme uses a point or line source directly, the multi-one-way scheme requires an initial boundary condition. To compare the results obtained with these two approaches, we need to transform a point/line source into a boundary condition, and vice versa. Following Collino and Joly (1999), a point/line source at depth 0, $f(x, 0)$, leads to the initial boundary condition $u(x, 0)$:

$$u(x, 0) = \frac{1}{2i} \Lambda_a^{-1} f(x, 0). \quad (17)$$

The initial boundary condition corresponds to the line source:

$$f(x, 0) = 2i \Lambda_a u(x, 0). \quad (18)$$

To apply least-squares migration with the multi-one-way scheme, we must transform the data, which are seen as point measurements, to an initial boundary condition using equation 17 before computing the back-propagated field (see Appendix C).

The modified imaging principle assumes the wavefield is measured on the domain boundary and the data correspond to a (continuous) measurement on the boundary. To apply it with the two-way scheme, a line source is defined from the data by applying equation 18 before computing the back-propagated field.

We should mention that equations 17 and 18 are derived assuming an interface between two homogeneous half-spaces. Therefore, around the interface — in our case, the acquisition line — the velocity should not vary too much. In marine acquisitions, the wavefield is measured close to the free surface. In this case, we should use deghosting techniques before applying the transformation.

NUMERICAL EXAMPLES

Fault model

The first example is synthetic marine data generated from a fault model (Figure 3) with an FD finite-difference code (Mulder and Plessix, 2004a). The modeling is acoustic with constant density. The data contain $N_s = 179$ shots of $N_r = 40$ receivers each. The spacing between sources and receivers is $\Delta = 25$ m. The position of the i th source is $(i - 1)\Delta$, and the position of the j th receiver of the i th source is $(i - 1)\Delta + j\Delta$. We use a smoothed version of the true model for migration. Our main goal is to select the best migration

weights with the thin-slab multi-one-way scheme. For completeness, we also present the result of the modified imaging principle.

In Figure 4, we display the vertical logs taken from the least-squares migrated images and the modified imaging principle result at $x = 4$ km. We calibrated the traces with the amplitudes of the first reflector because the preserved-amplitude migration formulas described in this article are correct up to a scalar, which depends on source wavelet and acquisition geometry. The type 1 and type 2 least-squares migrations (Figure 4a and b) overestimate the amplitude of the deepest reflector; the type 3 least-squares migration (Figure 4c) gives the best result. The amplitude estimate improves slightly with the modified imaging principle (Figure 4d).

In Plessix and Mulder (2004), where the wavefields are computed with a two-way solver and large offset data are used, the type 2 least-squares migration provides the best migrated images. Here, we consider only small-to-medium offset data because we use thin-slab multi-one-way modeling. The angle limitation and the absence of reflected and diving waves in the one-way scheme may explain why type 2 migration works better with two-way modeling and type 3 with thin-slab multi-one-way modeling. We display the migrated image obtained with type 3 least-squares migration and the one-way scheme in Figure 5.

2D SEG/EAGE salt model

The second example is based on a 2D section of the SEG/EAGE salt model (Figure 6). For the migration, we used a smoothed version (Figure 1). Considering a marine acquisition, 237 shots of 65 receivers are generated with a time-domain finite-difference code. The distance between two shots is 80 m, and the distance between two re-

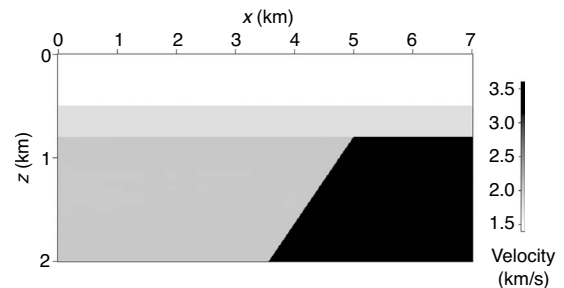


Figure 3. Acoustic velocity of the fault model in kilometers/second.

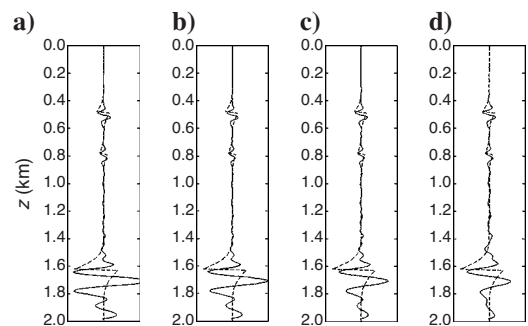


Figure 4. Comparison of the slowness contrasts at $x = 4$ km. The true contrasts correspond to the thick solid line, the dashed lines correspond to the retrieved contrasts. (a) Least-squares approach and type 1 preconditioner. (b) Least-squares approach and type 2 preconditioner. (c) Least-squares approach and type 3 preconditioner. (d) Modified imaging principle after stacking over shots and receivers.

ceivers is 40 m. Figure 7 shows the true slowness contrasts, equal to the difference between the smooth slowness model and the full slowness model divided by the smooth model. In the sediment zones, the amplitude contrasts are roughly identical at all of the discontinuities. We shall use this behavior to check the migrated images.

We consider four computations:

- 1) The type 3 least-squares migration with the wavefields computed with the thin-slab multi-one-way scheme (Figure 8a)
- 2) The type 2 least-squares migration with the wavefields computed with the two-way scheme (Figure 8b)
- 3) The modified imaging principle with the wavefields computed with the thin-slab multi-one-way scheme (Figure 8c)
- 4) The modified imaging principle with the wavefields computed with the two-way scheme (Figure 8d)

The least-squares migration with the thin-slab multi-one-way scheme overestimates the amplitudes in the sediment region at the right of the salt body (Figure 8a). Moreover, the amplitudes of the

flat reflectors beneath the salt body are not correctly recovered. Those amplitudes should be laterally constant, but they are weaker beneath the salt body. These unwanted artifacts almost disappear when the modified imaging principle is used (Figure 8c).

With the two-way scheme and least-squares migration (Figure 8b), the slowness contrasts in the sediment zones are estimated correctly except for part of the deepest reflector just beneath the salt body. The least-squares two-way migration displays the well-known diving-wave artifacts in the upper part of the image (Mulder and Plessix, 2004a). The image obtained with the two-way scheme and the modified image principle (Figure 8d) do not suffer from the diving-wave artifact because the latter is filtered out by the derivatives of the wavefields in equation 1.

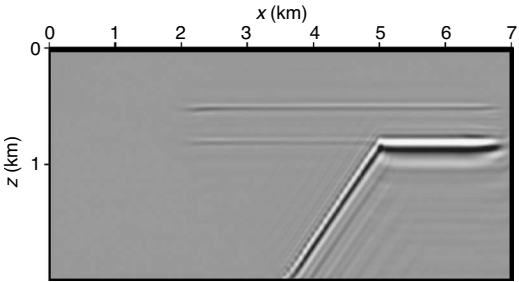


Figure 5. Migrated image of the fault model obtained with the least-squares approach and type 3 preconditioner.

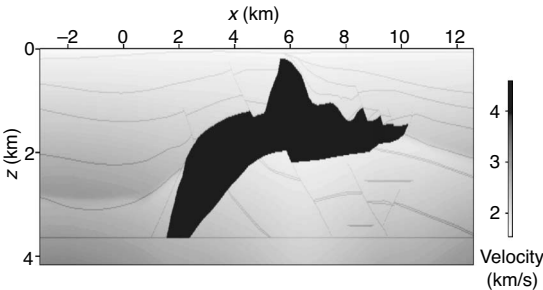


Figure 6. True acoustic velocity of the 2D section of the SEG/EAGE salt model in kilometers per second.

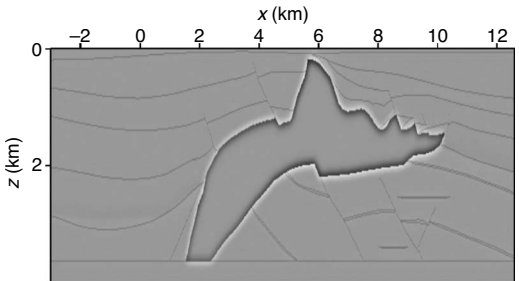


Figure 7. True slowness contrast of the salt model.

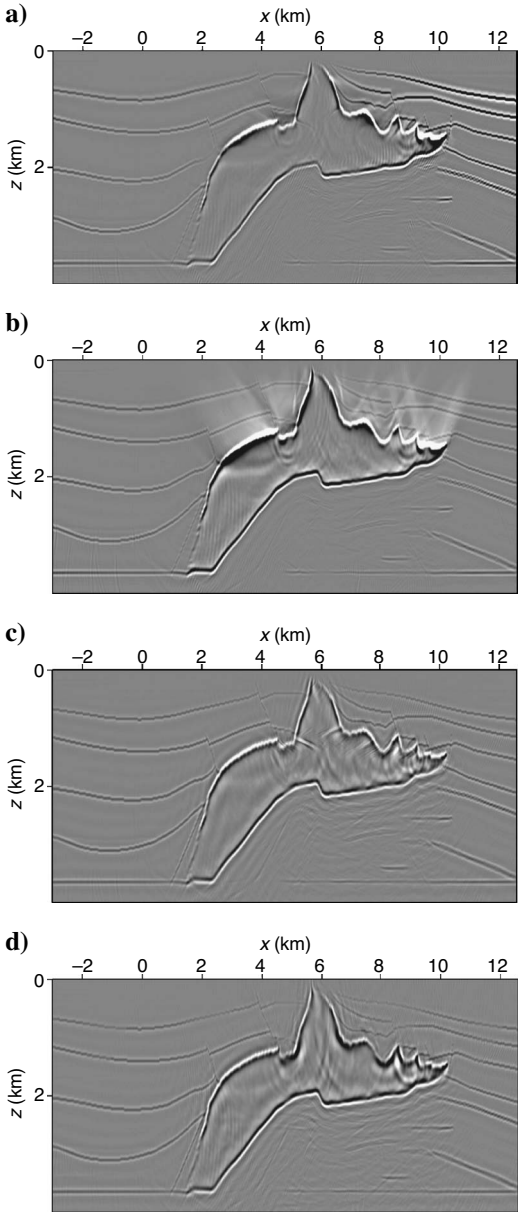


Figure 8. Migrated image of the salt model obtained with (a) type 3 least-squares migration and thin-slab multi-one-way scheme; (b) type 2 least-squares migration and two-way scheme; (c) modified imaging principle and thin slab multi-one-way scheme; and (d) modified imaging principle and two-way scheme.

The modified imaging principle seems to give the best amplitude estimates of the slowness contrasts. At least under the high-frequency approximation, this approach is similar to Bleistein's inversion approach (Kiyashchenko et al., 2007) and does not depend on a diagonal approximation of the Hessian. However, the images obtained

with the modified imaging principle are noisier in the area where energy defocusing exists, notably beneath the salt.

Also, the modified imaging principle introduces a number of artifacts in the salt. In those regions, least-squares migration provides cleaner images. In fact, least-squares migration is more stable because the stack is always done over the shots and receivers before any normalization: The prestack shot-based images are first stacked together and then are divided by the weighted stack of the wavefield amplitudes. With the modified imaging principle, the image is normalized by shot before the final stack, which makes it more sensitive to the presence of shadow zones. In practice, regularization is needed to reduce those artifacts.

We illustrate these findings in Figure 9 with some logs of the true slowness contrasts, of the two-way least-squares migrated image, and of the two-way modified imaging principle result. The amplitudes at the salt interfaces, between 1 and 2 km depth and 3 to 10 km on the horizontal axis, are incorrectly estimated in Figure 9b, whereas in Figure 9c they are closer to the true ones. However, the assumption of small contrasts breaks down at the salt interfaces. This prevents these two approaches from fully recovering the amplitudes at the salt interfaces. The amplitude of the deepest reflector, at 3.6 km depth, is more continuous with the modified imaging principle.

We also plotted the logs in the sediment zone at $x = -0.7$ km in Figure 10 and across the salt body at $x = 9.8$ km in Figure 11. To help the interpretation, we displayed a band-limited version of the true slowness contrasts using the approach described in Thierry et al. (1999). Again, we can see that the amplitudes in the sediment zones generally are recovered better with the modified imaging principle.

CONCLUSIONS

Contrary to the classic Claerhout imaging principle, which estimates angle-dependent reflection coefficients, the modified imaging principle, similarly to least-squares migration, estimates the slowness contrasts, which correspond, for the acoustic case with constant density, to a quantitative structural image. Numerical comparisons between least-squares migration and the modified imaging principle have shown that the latter recovers the amplitudes better but introduces more artifacts to the image. This trade-off should be taken into account when both methods are used.

The numerical results of this paper, based on a marine acquisition, illustrate that the modified imaging principle can be applied with a finite-difference wave-equation solver. The modified imaging principle does not rely on a diagonal approximation of the Hessian and therefore implicitly takes into account the off-diagonal terms of the Hessian. This explains why it estimates amplitudes better. However, the modified imaging principle relies on infinite coverage and is shot based. With limited coverage and the presence of shadow zones where the energy is defocused, artifacts and noise appear in the images. Least-squares migration is less sensitive to a shadow zone because all shots are handled together. Derivation of the least-squares migration weights and modification of the imaging principle assume small contrasts. This explains why the amplitudes at the salt interfaces are recovered incorrectly.

The migrated images obtained with the two-way or the thin-slab multi-one-way schemes are quite similar. In the numerical exam-

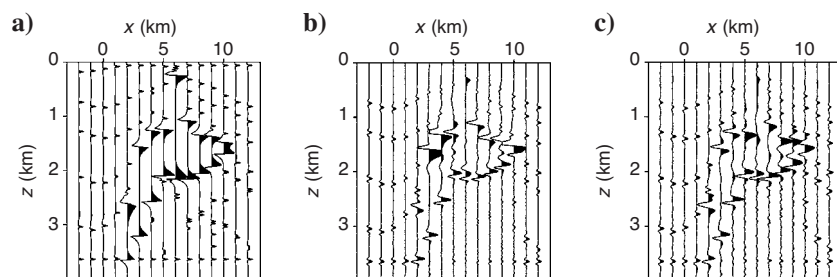


Figure 9. Some logs of (a) the true slowness contrast image, (b) the least-squares migrated image obtained with the two-way scheme, and (c) the modified imaging principle result.

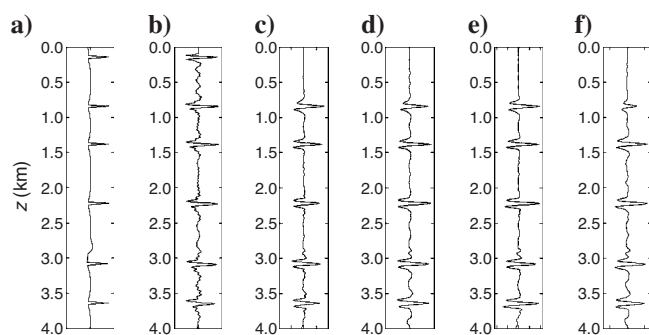


Figure 10. Comparison, in the sediment zone at $x = -0.7$ km, between (a) the true slowness contrasts, (b) the band-limited true slowness, (c) the retrieved contrasts with least-squares migration based on the thin-slab multi-one-way scheme, (d) with migration based on the modified imaging principle and the thin-slab multi-one-way scheme, (e) with least-squares migration based on the two-way scheme, and (f) with the modified imaging principle and the two-way scheme.

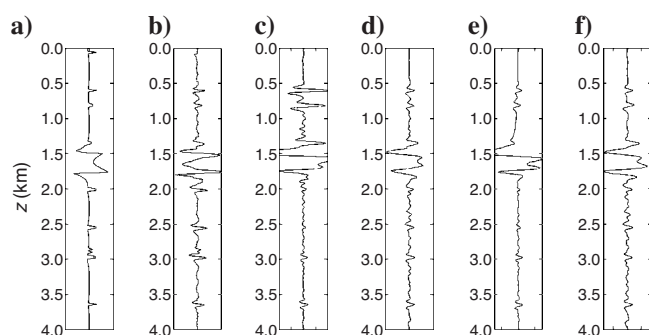


Figure 11. Comparison, through the salt at $x = 9.8$ km, between (a) the true slowness contrasts, (b) the band-limited true slowness, (c) the retrieved contrasts with least-squares migration based on the thin-slab multi-one-way scheme, (d) with migration based on the modified imaging principle and the thin-slab multi-one-way scheme, (e) with least-squares migration based on the two-way scheme, and (f) with the modified imaging principle and the two-way scheme.

ples, we used only short-to-medium offsets. These results show the relevance of the thin-slab multi-one-way scheme to estimate the amplitudes of the wavefields. Larger differences are expected at larger offsets. The well-known diving-wave artifacts of two-way migration are not present in the two-way modified imaging principle because of the derivatives of the wavefields in formula 1.

Our approaches are valid with a purely acoustic-wave propagation with constant density and small velocity contrasts. The modified imaging principle should be adapted further to be applied to long-offset data where the amplitudes also depend on shear-velocity contrasts.

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APPENDIX A

MODIFIED IMAGING PRINCIPLE: SKETCH OF THE DERIVATION

The classic imaging principle (Claerbout, 1971) defines the common-shot image by

$$I_C(\mathbf{x}_s, \mathbf{x}) = \frac{1}{A(\mathbf{x}_s, \mathbf{x})} \int_{\omega_{\min}}^{\omega_{\max}} d\omega u_d(\mathbf{x}_s, \mathbf{x}, \omega) \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega), \quad (\text{A-1})$$

with

$$A(\mathbf{x}_s, \mathbf{x}) = \int_{\omega_{\min}}^{\omega_{\max}} d\omega u_d(\mathbf{x}_s, \mathbf{x}, \omega) \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega). \quad (\text{A-2})$$

Here, $u_d(\mathbf{x}_s, \mathbf{x}, \omega)$ is the incident wavefield generated by the source at $\mathbf{x}_s$, and $u_b(\mathbf{x}_s, \mathbf{x}, \omega)$ is the back-propagated wavefield of the shot gather $d(\mathbf{x}_s, \mathbf{x}, \omega)$ recorded at the earth's surface. The value A is the norm of the incident field.

In equation A-1, $I_C(\mathbf{x}_s, \mathbf{x})$ approximates the plane-wave reflection coefficients, and it is shot dependent. For small velocity contrasts and small to medium incident angles θ , the reflection coefficient at a given interface $R(\theta)$ can be approximated by the relative slowness contrast divided by $\cos^2 \theta$ (Bleistein et al., 2001). A multiplication by $\cos^2 \theta$ of $\bar{u}_b u_d$ inside the integral approximates this contrast. When the modeling is based on a finite-difference solution of the wave equation, the reflection angle θ is unknown. However, we can use $2 \cos^2 \theta = 1 + (\sin \alpha \sin \beta + \cos \alpha \cos \beta)$, with α (respectively, β) the angle between the vertical and the slowness vector of the incident (respectively, back-propagated) plane wave at the subsurface point.

The sine and cosine of those angles correspond to the horizontal and vertical wavenumbers divided by the angular frequency. Formally, the horizontal wavenumber is obtained by applying $(1/ik) \times (\partial/\partial x)$ to the wavefield, and the vertical wavenumber is obtained by applying Λ_a to the wavefield. The modified imaging principle is obtained by multiplying the integrand of the classic imaging principle by $\cos^2 \theta$ and by rewriting $\cos^2 \theta$ in terms of quantities computed with the one-way scheme. This gives

$$\begin{aligned} \tilde{I}_M(\mathbf{x}_s, \mathbf{x}) = & \frac{1}{A(\mathbf{x}_s, \mathbf{x})} \int_{\omega_{\min}}^{\omega_{\max}} d\omega \left(u_d(\mathbf{x}_s, \mathbf{x}, \omega) \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega) \right. \\ & + \frac{1}{k^2} \left[\frac{1}{i} \frac{\partial u_d(\mathbf{x}_s, \mathbf{x}, \omega)}{\partial x} \frac{1}{i} \frac{\partial \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega)}{\partial x} \right. \\ & \left. \left. + \Lambda_a u_d(\mathbf{x}_s, \mathbf{x}, \omega) \Lambda_a \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega) \right] \right), \quad (\text{A-3}) \end{aligned}$$

The value $\tilde{I}_M$ approximates the slowness jumps around the interface but slightly differs from the band-limited slowness contrast. The band-limited slowness contrast is obtained by taking into account the frequency content of the data (Kiyashchenko et al., 2007):

$$\begin{aligned} I_M(\mathbf{x}_s, \mathbf{x}) = & \frac{\int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{|\omega i|}}{\pi A(\mathbf{x}_s, \mathbf{x})} \int_{\omega_{\min}}^{\omega_{\max}} d\omega \operatorname{sgn} \omega \left(u_d(\mathbf{x}_s, \mathbf{x}, \omega) \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega) \right. \\ & - \frac{1}{k^2} \left[\frac{\partial u_d(\mathbf{x}_s, \mathbf{x}, \omega)}{\partial x} \frac{\partial \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega)}{\partial x} \right. \\ & \left. \left. - \Lambda_a u_d(\mathbf{x}_s, \mathbf{x}, \omega) \Lambda_a \bar{u}_b(\mathbf{x}_s, \mathbf{x}, \omega) \right] \right). \quad (\text{A-4}) \end{aligned}$$

This formula is equivalent to the pseudoinverse formula 5.1.10 of Bleistein et al. (2001).

APPENDIX B

COVERAGE LIMITATION EFFECTS

In this appendix, we explain why the stack of the common-shot and common-receiver images improves the amplitude estimates.

A marine acquisition system consists of a source and a receiver line of length L ; L corresponds to the maximum offset. This system is towed along the acquisition line.

Assuming a homogeneous background, the energy reflected by a subsurface point $\mathbf{x}$ belonging to a reflector is recorded only if the source point S belongs to the segment $[S_0(\mathbf{x}), S_1(\mathbf{x})]$ (see Figure B-1). The term $S_0(\mathbf{x})$ corresponds to the source point when the energy is recorded by the receiver $R_0(\mathbf{x})$, located at $S_0(\mathbf{x})$. This is the zero-offset

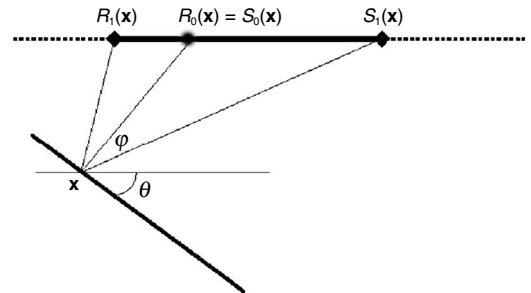


Figure B-1. Simple seismic experiment. Here, $\mathbf{x}$ is an image point; $|R_1(\mathbf{x})R_0(\mathbf{x})|$ is the receiver line; $|S_0(\mathbf{x})S_1(\mathbf{x})|$ is the source line seen by $\mathbf{x}$; $S_0(\mathbf{x}) = R_0(\mathbf{x})$ is the source and receiver position of the zero-offset reflection; θ is the dip angle; and φ is the incidence angle.

case ($R_0 = S_0$). The term $S_1(\mathbf{x})$ corresponds to the source point when the energy is recorded by the receiver $R_1(\mathbf{x})$, located at the maximum offset. The length $|S_0(\mathbf{x})S_1(\mathbf{x})|$ depends on $\mathbf{x}$ and, more precisely, on the dip angle of the reflector where $\mathbf{x}$ sits, as shown in Figure B-1. (For flat reflectors and in a homogeneous background, $|S_0(\mathbf{x})S_1(\mathbf{x})| = L/2$ and is independent of $\mathbf{x}$.) In the common-shot image I_M with a finite coverage, the amplitude of the slowness contrast at $\mathbf{x}$ is multiplied approximately by $|S_0(\mathbf{x})S_1(\mathbf{x})|$. When $|S_0(\mathbf{x})S_1(\mathbf{x})|$ largely depends on $\mathbf{x}$, which is the case with large lateral velocity variations, coverage effects are present in the common-shot image $\Sigma_{\mathbf{x}} I_M(\mathbf{x}, \mathbf{x})$.

If the data are resorted by receivers in the common-receiver image $\Sigma_{\mathbf{x}} I_M(\mathbf{x}, \mathbf{x})$, the amplitude of the slowness contrast at $\mathbf{x}$ is approximately multiplied by $|R_1(\mathbf{x})R_0(\mathbf{x})|$. The term $R_0(\mathbf{x})$ corresponds to the receiver point, seen as a source point, where the energy is recorded by the source (seen as a receiver) located at $S_0(\mathbf{x})$. The term $R_1(\mathbf{x})$ corresponds to the receiver point, seen as a source point, where the energy is recorded by the source $S_1(\mathbf{x})$, seen as a receiver point located at the maximum offset. Because $S_0(\mathbf{x}) = R_0(\mathbf{x})$, $|R_1(\mathbf{x})S_1(\mathbf{x})| = |R_1(\mathbf{x})R_0(\mathbf{x})| + |S_0(\mathbf{x})S_1(\mathbf{x})| = L$ the stack of the shot-based and receiver-based images is independent of $\mathbf{x}$. Therefore, stacking the common-shot and common-receiver images reduces the influence of the coverage.

Notice that this difficulty occurs because we are working with a common-shot formula. It would disappear if a common-image-point formula could be applied. However, working with common-mid-point gathers is too expensive with a finite-difference modeling of the wave equation.

APPENDIX C

BACK-PROPAGATED WAVEFIELDS COMPUTATION

We obtain the multi-one-way equations for a back-propagated field by taking the conjugate of equation 10 where $\bar{u}_b = u_b$:

$$\left(\frac{\partial}{\partial z} - i\Lambda_a\right)\left(\frac{\partial}{\partial z} + i\Lambda_a\right)u_b = d_f + \bar{E}u_b, \quad (\text{C-1})$$

where d_f is the equivalent of the source f of the forward modeling. Here, d_f corresponds to the measurement of the wavefield at the receiver locations.

The thin-slab multi-one-way method consists of three steps to compute the back-propagated wavefield $u_b \approx u_{b0} + u_{b1}$ (where an overbar denotes the conjugate):

- 1) One step down. Solve from z and $z + \Delta z$:

$$\frac{\partial u_{b0}}{\partial z} = -i\Lambda_a u_{b0}, \quad (\text{C-2})$$

with $u_{b0}(z) = u(z)$.

- 2) One step up. Solve from $z + \Delta z$:

$$\left(\frac{\partial}{\partial z} - i\Lambda_a\right)u_{b1/2} = \bar{E}u_{b0}, \quad (\text{C-3})$$

with $u_{b1/2}(z + \Delta z) = 0$.

- 3) One step down.

$$\left(\frac{\partial}{\partial z} + i\Lambda_a\right)u_{b1} = u_{b1/2}, \quad (\text{C-4})$$

$$\text{with } u_{b1}(z) = \frac{1}{2i}\Lambda_a^{-1}u_{b1/2}(z).$$

As for the forward multi-one-way scheme, an initial boundary condition is required.

Considering the point source f and the data d , the different approaches can be summarized as follows:

- In two-way modeling and least-squares migration, the modeling requires point/line sources and the data are interpreted as point measurements; therefore, no transformations of f and d are required.
- In two-way modeling and the modified imaging principle, the modeling requires point/line sources and the data are interpreted as a continuous measurement on the boundary; therefore the data d must be transformed to a line source using equation 17.
- In multi-one-way modeling and the modified imaging principle, the modeling requires initial boundary conditions and the data are interpreted as a continuous measurement on the boundary; therefore, only the source point f must be transformed to an initial boundary condition using equation 18.
- In multi-one-way modeling and least-squares migration, the modeling requires initial boundary conditions and the data are interpreted as point measurements; therefore, f and d need to be transformed into boundary conditions using equation 18.

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