

Influence of matric suction on shear strength behavior of a residual clayey soil

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Abstract In this paper, the shear strength with respect to the matric suction of unsaturated soils was studied. For this purpose, unsaturated triaxial testing procedures were applied to the undisturbed residual soil specimens. An apparatus for performing triaxial tests was designed and constructed. In the tests, matric suction was controlled by using the axis translation technique, and pore water volume changes were measured by means of a volume change transducer with 10^{-8} m^3 sensitivity. The test results indicated that the matric suction contributes to the shear strength of unsaturated soil specimens, and this contribution called suction strength varies non-linearly with respect to the matric suction. The logarithmic model needing to know the air-entry value and the internal friction angle of a soil specimen for prediction of the suction strength were presented and compared with the test results. It was found that suction strength values predicted from the proposed model were in satisfactory agreement with the experimental results.

Keywords Unsaturated soil · Matric suction · Shear strength · Residual clay

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List of symbols

c'	effective cohesion
C_T	total cohesion
ϕ'	effective friction angle
τ_f	shear strength
τ_{us}	suction strength
P_{at}	atmospheric pressure
θ	volumetric water content
θ_s	saturated volumetric water content
θ_r	volumetric water content at residual condition
S_r	degree of saturation
$(u_a - u_w)$	matric suction
$(u_a - u_w)_b$	air-entry suction of soil
$(u_a - u_w)_r$	residual suction of soil
ψ	soil suction

Introduction

The stress states in soil consist of certain combinations of stress variables that can be referred to as stress state variables. Stress state is independent of the physical properties of a soil. The number of stress state variables depends on the number of the phase involved. Unsaturated soils are characterized by the presence of air phase, water phase and air–water interface in voids. It is thus difficult to define convenient stress state variables for unsaturated soils. For practical engineering purposes, these variables of unsaturated soils are generally the net stress: $(\sigma - u_a)$, and matric suction: $(u_a - u_w)$. The matric suction is adopted as an independent stress state variable, so that the use of a single-valued effective stress equation for unsaturated soil can be eliminated (Fredlund and Rahardjo 1993). This approach is

commonly adopted by most of the researchers studying unsaturated soils (Vanapalli et al. 1994; Gan and Fredlund 1996; Miao et al. 2002; Toll and Ong 2003). The shear strength of unsaturated soils is, therefore, defined by using these two stress state variables. So the effect of matric suction on shear strength is defined. As the soils are de-saturated, soil stiffness rises due to the decrease in water content of soil specimens or the increase in matric suction. This provides a contribution to the shear strength of unsaturated soils. Attempts indicate that this contribution called the suction strength exhibits a non-linear behavior with matric suction (Oloo and Fredlund 1996; Oberg and Sallfors 1997; Miao et al. 2002; Xu 2004).

The shear strength tests of unsaturated soils are carried out by using modified triaxial and direct shear equipment to allow for control of the matric suction by using the axis translation technique (Coleman 1962; Bishop and Blight 1963; Satija 1978; Ho and Fredlund 1982; Wulfsohn et al. 1998; Wong et al. 2001; Aversa and Nicotera 2002). However, these laboratory tests for unsaturated soils are fairly complicated, time consuming and expensive. Hence, some investigators dealing with unsaturated soils suggest empirical approaches for the prediction of suction strength.

In this paper, the shear strength with respect to the matric suction of a residual soil sample taken from the Diyarbakir region in the southeast of Turkey is investigated. For this aim, the triaxial test apparatus was modified to measure the shear strength parameters of unsaturated soils. The tests in which undisturbed soil specimens were used were performed by using an axis translation technique to control the matric suction. The logarithmic model for the prediction of the suction strength was presented and compared with the test results. Furthermore, some suction strength equations from the available literature were comparatively reviewed.

Review of shear strength of unsaturated soils

Suction strength equations of unsaturated soils

One of the well-known shear strength equations for unsaturated soils was proposed by Bishop (1959) as shown below.

$$\tau = c' + [(\sigma - u_a) + \chi(u_a - u_w)] \tan \phi' \quad (1)$$

He extended Terzaghi's principle of effective stress for saturated soils. In this equation, $(u_a - u_w)$ is matric suction, and the χ parameter is dependent on the degree of saturation. χ has a magnitude between 1 and 0 and represents the variation of condition from fully saturated to total dryness. Note that Bishop's equation consists of two parts. The first

part of these equations is related to saturated shear strength when the pore-air pressure, u_a , is equal to the pore-water pressure, u_w . This part is a function of normal stress, since the shear strength parameters c' and ϕ' are constant for a saturated soil. The second part of this equation is the contribution of matric suction, which is called suction strength (τ_{us})

$$\tau_{us} = \chi(u_a - u_w) \tan \phi' \quad (2)$$

Another suction strength equation was defined by Fredlund et al. (1978) in terms of independent stress state variables as shown below:

$$\tau_{us} = (u_a - u_w) \tan \phi^b \quad (3)$$

ϕ^b is the contribution of the matric suction to shear strength. According to the earlier limited studies performed at the low range of suction values available in the literature, this contribution was assumed to be linear. However, later experimental studies have shown that a non-linear increase in shear strength exists with respect to the matric suction (Gulhati and Satija 1981; Escario and Saez 1982; Gan and Fredlund 1982; Escario and Juca 1989; Fredlund et al. 1996; Oloo and Fredlund 1996; Rassam and Williams 1997). This non-linearity was explained by Oloo and Fredlund (1996) as being the result of the diminishing contribution of the matric suction to the shear strength as the water content of the soil approaches the residual water content.

The relationship between shear strength of unsaturated soils and soil water characteristic curve

The soil water characteristic curve (SWCC) describes the relationship between the soil suction and volumetric water content, θ , or the degree of saturation (S_r). The SWCC is widely used to define the behavior of unsaturated soils. In recent years, some researchers have reported empirical procedures to predict the shear strength of unsaturated soils by using the SWCC and saturated shear strength parameters.

Vanapalli et al. (1996) developed a physical model for explaining the unsaturated shear strength behavior. They identified the different saturation stages when the desaturation process of soil takes place. According to this model, the soils have three identifiable stages during desaturation: the boundary stage, the transition stage (i.e., primary and secondary transition stage) and residual stage as shown in Fig. 1. The soil is essentially saturated in the boundary stage. The area of water in the soil does not change with the increasing matric suction. The variation of shear strength is assumed to be linear. Hence, angle ϕ^b is equal to the

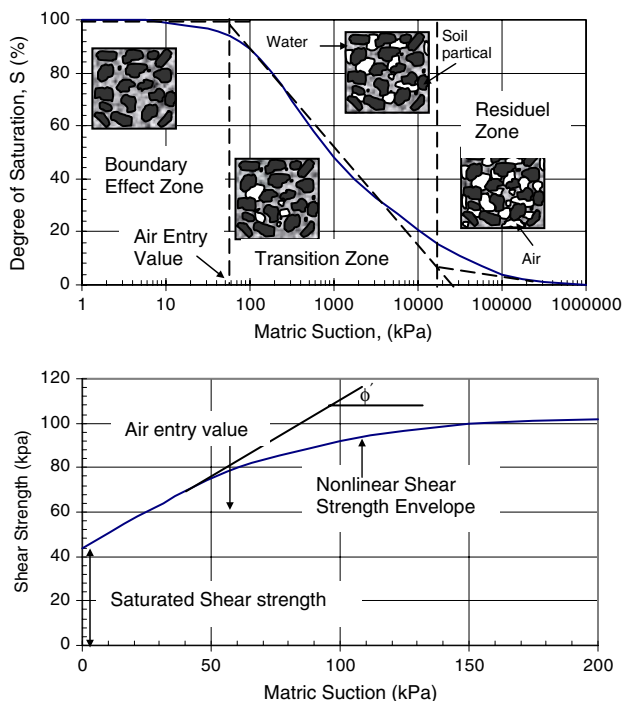


Fig. 1 **a** A typical soil water characteristic curve showing desaturation stage. **b** The relationship between shear strength behavior of soil and the soil water characteristic curve (Vanapalli et al. 1996)

effective internal frictional angle up to the air entry value of soil. In the transition stage, the matric suction is greater than the air entry value. The water content reduces with the increasing matric suction. Namely, the air starts to enter into the soil and the shear strength exhibits a non-linear behavior. In the residual stage, the variation of water content changes is fairly small. The water content that soils have at this stage is generally referred to as the residual water content. Beyond this stage, shear strength with respect to the matric suction decreases or remains relatively constant for sandy and silty soils. However, an increment in the shear strength can be observed for clayey soils (Vanapalli et al. 1996).

Lamborn (1986) suggested an equation for predicting the suction strength by using a micro mechanics model as follows:

$$\tau_{us} = (u_a - u_w)\theta \tan \phi' \tag{4}$$

where τ_{us} is suction strength and θ volumetric water content (i.e., the ratio of the volume of water to the total volume of the soil). Vanapalli et al. (1996) proposed the following model for predicting the suction strength by using the soil–water characteristic curve:

$$\tau_{us} = (u_a - u_w) \tan \phi' \left[\frac{\theta - \theta_r}{\theta_s - \theta_r} \right] \tag{5}$$

in which θ_s is saturated volumetric water content, θ_r volumetric water content at residual conditions and $(u_a - u_w)$ matric suction. This model requires the use of the entire part of the SWCC curve. Since the clays do not have a well-defined residual state, the determination of θ_r is difficult.

Another equation for suction strength, which is similar to that of Bishop (1959), was proposed by Oberg and Sallfors (1997) for sands and silts:

$$\tau_{us} = (u_a - u_w)(S_r) \tan \phi'. \tag{6}$$

The authors assumed that the χ -factor is equal to the degree of saturation, S_r . The variation of the χ -factor with matric suction was investigated by Khalili and Khabbaz (1998) by using experimental data published in the literature. The χ -factor was proposed as $\chi = [(u_a - u_w)/(u_a - u_w)_b]^{-0.55}$. By replacing the χ -factor in Bishop’s equation, suction strength, τ_{us} , is given as:

$$\tau_{us} = (u_a - u_w) \left[\frac{(u_a - u_w)}{(u_a - u_w)_b} \right]^{-0.55} \tan \phi' \tag{7}$$

where $(u_a - u_w)_b$ is the air entry value of soils. It can be seen that this equation is used for all types of soils. Another equation relating the suction strength to the soil–water characteristic curve was put forward by Rassam and Cook (2002) as follows:

$$\tau_{us} = (u_a - u_w) \tan \phi' - \varphi [(u_a - u_w) - (u_a - u_w)_b]^\beta \tag{8}$$

$$\varphi = \frac{(u_a - u_w)_r \tan \phi' - \tau_{sr}}{[(u_a - u_w)_r - (u_a - u_w)_b]^\beta} \tag{9}$$

$$\beta = \frac{\tan \phi' [(u_a - u_w)_r - (u_a - u_w)_b]}{(u_a - u_w)_r \tan \phi' - \tau_{sr}} \tag{10}$$

where φ and β are estimated fitting parameters, $(u_a - u_w)_r$ is residual suction (i.e., matric suction of the soil specimen having residual water content) and the τ_{sr} contribution of matric suction to the shear strength at residual suction, which should be experimentally evaluated. This method is most suitable for coarse- to medium-grained soils where the residual suction (τ_{sr}) may be achieved reliably in the laboratory.

Empirical equations to represent the variation of shear strength with respect to suction

Determination of the parameters in the above-mentioned equations, such as θ_r , θ_s , $(u_a - u_w)_b$, $(u_a - u_w)_r$ and τ_{sr} , involves a sophisticated and difficult testing program. Hence, several investigators have reported different

empirical equations to represent the variation of shear strength with respect to suction. One of these equations, which is an elliptical curve of degree 2.5, was given by Escario and Juca (1989):

$$\left(\frac{s_m - \Psi}{a}\right)^{2.5} + \left(\frac{\tau - \tau_b}{b}\right)^{2.5} = 1 \quad (11)$$

where Ψ is soil suction, s_m , τ_b , a and b are soil parameters. Shen and Yu (1996) have presented a hyperbolic equation for the shear strength of unsaturated soils as shown below.

$$\tau_{us} = \frac{\psi}{1 + \psi d} \tan \phi' \quad (12)$$

where ψ is soil suction and d is the fitting coefficient. This equation enables one to predict the suction strength for a large range of suction values by using experimental results for a small range of suction values.

Another hyperbolic model has been suggested by Miao et al. (2002). In this model, the non-linear relationship between the suction strength and matric suction is expressed in $[1/(u_s + P_{at}), 1/(\tau_{us} + P_{at})]$ plane as a straight line.

$$\tau_{us} = \frac{a(u_a - u_w)}{1 + \frac{(1-a)}{P_{at}}(u_a - u_w)} \quad (13)$$

in which a is a soil parameter, and P_{at} is the atmospheric pressure.

Testing apparatus

The stress–strain behavior of unsaturated soils is interpreted from results of triaxial or direct shear tests. These tests are generally performed by controlling matric suction (Aversa and Nicotera 2002; Alonso et al. 1990; Fredlund and Rahardjo 1993). The matric suction in the soil specimen is controlled by using the axis translation technique. In this technique, the pore-air pressure is artificially raised above atmospheric pressure to increase the pore water pressure by the same amount to a positive value. In this way, the cavitation risk of water in the measuring system is prevented (Fredlund and Rahardjo 1993). In this study, the triaxial test apparatus was modified to perform the unsaturated shear strength tests. The apparatus is capable of controlling as well as measuring pore air and pore water pressure in the soil specimen independently by using the axis translation technique. Pore-water pressure was measured and controlled through a saturated ceramic disk having 500 kPa of air entry value integrated with the above pedestal connected to the measuring system water compartment. For unsaturated soils, the total volume change is

equal to the sum of the water and air volume changes. Due to the difficulty in measuring air-volume changes, the overall volume change and pore-water-volume change of the soil specimen are generally measured. The air volume change is calculated as a difference between the overall and pore water volume change (Fredlund and Rahardjo 1993). In this work, the water content variation was measured via a volume change transducer connected to the water compartment below the high air entry ceramic disc. The volume change transducer allows continuous electrical monitoring of the volume change of pore water with 10^{-8} m^3 sensitivity. Another volume change transducer was connected between the constant pressure device and triaxial cell to measure the overall volume change of soil specimens. The air applied on soil specimens can diffuse into cell water through the rubber membrane. This leads to an error in measurement of the volume change. In order to prevent the diffusion of the air, the soil specimens were enclosed in two rubber membranes with two slotted aluminum sheets separated by a layer silicon grease between the membranes as suggested by Alonso et al. (1990).

The system arrangement is shown in Fig. 2. The system includes a triaxial cell, pressure–volume controllers, plumbing arrangements and data acquisition system. The system enables computer-controlled stress or strain rate testing and can give real-time graphical outputs.

The modified triaxial cell and base plate are shown in Fig. 3. The base plate has four valves. Valve A in the base plate is used to measure and control pore-water pressure and water content change during the drained tests. Valve A is connected to volume change transducer. Valve B is used as a flushing system connecting the diffused air volume indica-

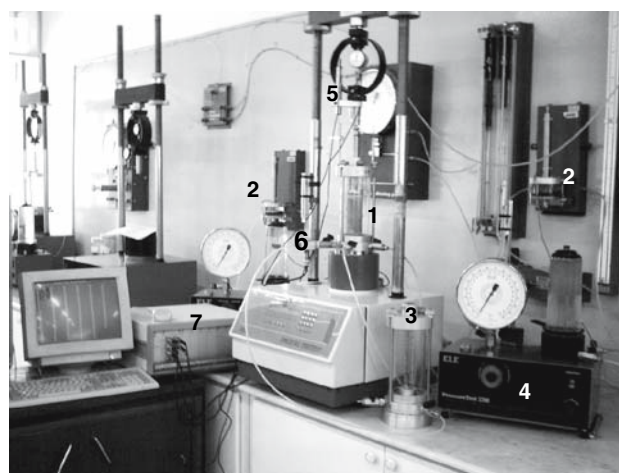
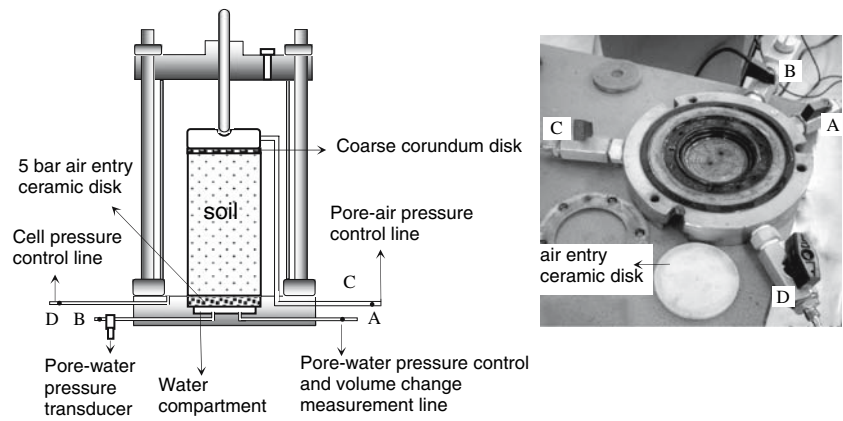


Fig. 2 Unsaturated triaxial test apparatus: 1 triaxial cell; 2 volume change transducer for pore-water and total volume change; 3 diffused air volume indicator; 4 constant pressure device; 5 vertical displacement transducer; 6 pore-water pressure transducer; 7 data logger

Fig. 3 Modified triaxial cell and base plate



tor. Valve C is connected to the pore-air pressure control or measurement line. The cell pressure is controlled or measured by means of valve D. The grooves inside the water compartment work as water channels for flushing air bubbles that have accumulated as a result of diffusion. The high-air entry ceramic disk (Soil Moisture Equipment Corporation, Santa Barbara, CA) with 500 kPa of air entry values is sealed into the pedestal using epoxy resin along its periphery. The air bubbles that accumulated in the water compartment as a result of diffusion were periodically removed by flushing. Therefore, the diffused air volume change was measured by using a diffused air volume indicator (DAVI) proposed by Fredlund and Rahardjo (1993), and the measurement of pore-water volume change was corrected.

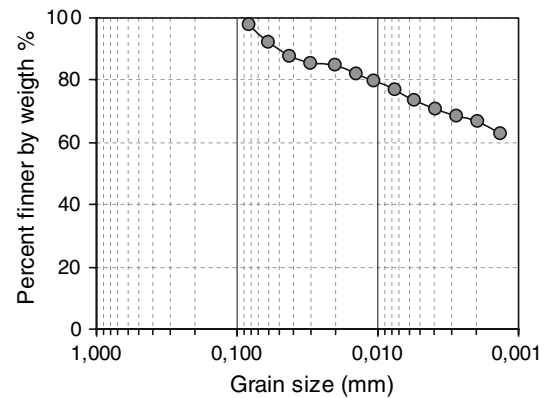


Table 1 Geotechnical properties of soil

Liquid limit (%)	77
Plastic limit (%)	32
Pasing #200 sieve (%)	95
<0.001 mm (%) (colloidal content)	55
Activity	0.79
Volumetric shrinkage (%)	9.5
Dry unit weight (kN/m ³)	14.81

technique as described by Fredlund and Rahardjo (1993). So the soil specimens were desaturated by removing the pore water from the soil specimens before shearing, and it was provided that the soil specimens had a certain matric suction value. The shear strength tests were performed on both saturated and unsaturated soil specimens, which have different matric suctions ranging from 50 to 400 kPa. The soil specimens were consolidated under a prescribed confining pressure of σ_3 , while the pore-air and pore-water pressures were controlled at a pressure of u_a and u_w , respectively. The overall volume change of the specimen was continuously monitored by means of a volume change transducer connected between the triaxial cell and constant pressure device. The volume of water flowing out of specimen was recorded by using another volume change transducer. When there is no longer a tendency for the overall volume change and the flow of water from the specimen, it was assumed that the consolidation reached an equilibrium condition.

Upon obtaining a stress and matric suction equalization under applied pressure (i.e., σ_3 , u_a and u_w), the soil specimens were sheared by compression at a strain rate of 0.004%/min. This strain rate is sufficiently slow to ensure fully drained conditions during shearing. The overall and water volume changes were recorded throughout the shearing. However, water volume changes have errors due to diffused air volume into the water compartment below the base plate. The air bubbles accumulated in the water compartment were periodically removed by flushing. The flushing was carried out by applying a water pressure from the constant pressure device to the water compartment. Diffused air volume was measured by flushing into the diffused air volume indicator, and water volume changes are corrected.

Test results

In both saturated and unsaturated triaxial tests, the undisturbed soil specimens, 50 mm in diameter and 100 mm in height, were tested. All specimens were prepared at these dimensions (i.e., length to diameter ratio L/D of 2) in order

to minimize the effects due to end platens of the apparatus and to reduce the likelihood of buckling during testing. For saturated soil specimens, shear strength parameters were measured from consolidation-drained tests, internal friction angle, ϕ' , of 21.9° and an effective cohesion, c' , of 14.82 kPa.

In the unsaturated triaxial tests, 12 undisturbed specimens having similar physical properties were used. The tests were conducted in accordance with the consolidated drained triaxial testing procedure. The matric suction was in the range 50–400 kPa in the experiment. This range is adequate for the practicing engineer, since engineers are generally concerned with the performance of geotechnical structures in the relatively low suction range of 0–500 kPa.

Figure 5 illustrates the stress–strain curves and variation of volumetric strain with axial strain for the saturated specimens under net confining pressures from 50 to 200 kPa. Stress–strain curves have a typical three-stage pattern for the change in deviatoric stress with axial strain. The deviatoric stress rapidly rises at a short axial strain range in the first stage. In the following stage, a wide range of constant rates of change in deviatoric stress with axial strain is shown. The third stage indicates greatly reduced change in deviatoric stress with axial strain. Figure 6 shows the deviatoric stress–strain relationship for all unsaturated specimens under net confining pressure ($\sigma_3 - u_a$) from 50 to 300 kPa. It appears that most of the speci-

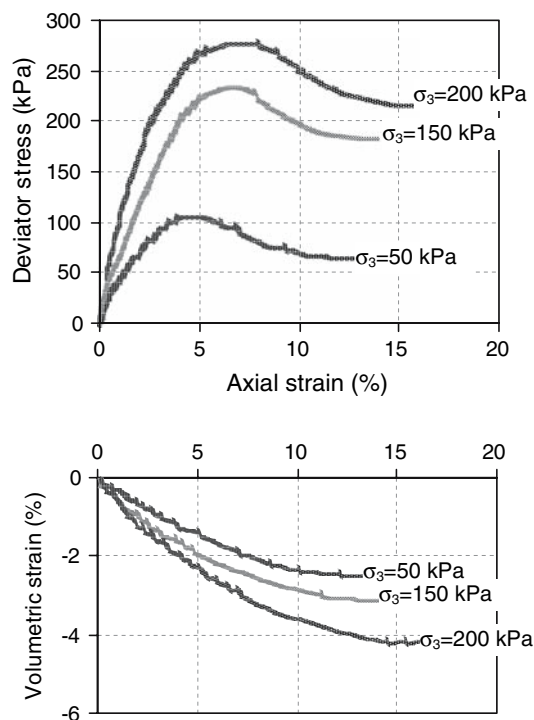
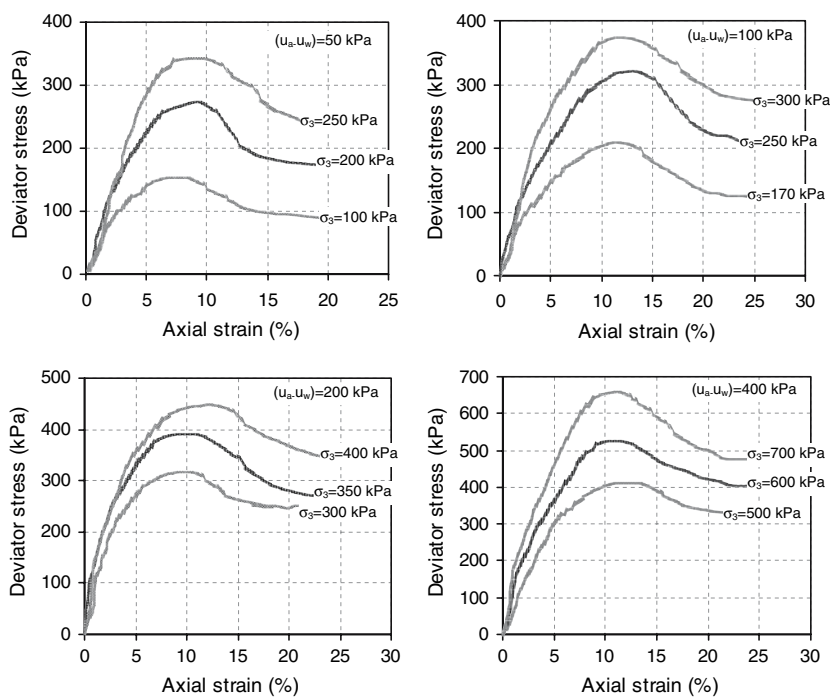


Fig. 5 Variation of stress and volumetric strain with axial strain for saturated soil specimen

Fig. 6 Stress versus strain relationship for unsaturated soil specimens



men are sheared at around 11% axial strain and a general pattern of these curves is similar to those for saturated specimens. It was observed that the matric suction has considerable effects on stress–strain curves. As can be seen in Fig. 6, the peak shear strength increases as the matric suction increases. This shows that the matric suction contributes to the shear strength of unsaturated soils. However, the general shape of the stress–strain curves is similar to those of saturated specimens. The variations of volumetric strain with axial strain during the shearing stage for all unsaturated specimens are demonstrated in Fig. 7. It appears that the total volume change of specimens becomes

stable at certain values of strain. This behavior shows that the soil specimens have a tendency to critical state condition as defined by Toll and Ong (2003) as the state achieved by a soil when it exhibits no changes in volume when it is sheared. The void ratio and degree of saturation of specimens at failure are also given in Table 2.

In Table 3, unsaturated triaxial test data were sorted out. Where $c_{total} = c' + \tau_{us}$, τ_{us} is suction strength. Figure 8 illustrates the relationship between the shear strength, τ , and net normal stress $(\sigma - u_a)$. It is seen that the unsaturated shear strength envelopes are approximately parallel to the saturated shear strength envelope and the effective cohe-

Fig. 7 Variation of volumetric strain with axial strain for unsaturated soil specimens

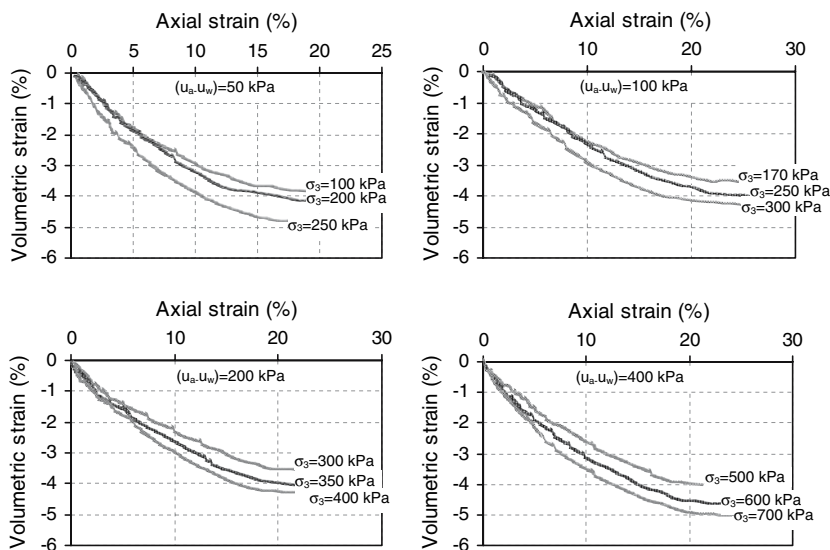


Table 2 The void ratio and degree of saturation of soil specimens at failure

u_a	50	50	50	100	100	100	200	200	200	400	400	400
σ_3	100	200	250	170	250	300	300	350	400	500	600	700
e	1.053	1.011	0.978	1.062	1.03	0.99	1.05	1.03	1.012	1.046	1.01	0.98
S_r	93.42	94.1	94.3	90.2	91.3	91.8	81	81.78	82.2	74.2	75	75.4

sion, c' , and effective frictional angle, ϕ' , are invariable. In other words, c' and ϕ' are independent of suction. Identical experimental results were reported by Drumright and Nelson (1995), Rohm and Vilar (1995) and Miao et al. (2002).

SWCC of soil used for this research

The SWCC of soil used for this research was determined from the pressure plate test. This test can be applied to determine the SWCC up to 1500 kPa. Entire SWCC (i.e., 0–1,000,000 kPa) was obtained by using the equation proposed by Fredlund and Xing (1996). Figure 9 shows the best-fit SWCC. The air entry value of soil was found to be approximately 40 kPa, and parameters θ_s and θ_r were determined to be 58.1 and 5.4%, respectively, from Fig. 9.

An equation for suction strength

Several investigators reported that the shear strength due to suction is non-linear. In this study, the variation of shear

strength with respect to the matric suction was derived by matching experimental data with a matching function. In order to obtain the approximate linear equation for the suction strength envelope, the relationship between the suction strength and matric suction was described in the $[\tau_{us}, \ln(u_a - u_w)]$ plane. In this case, the suction strength envelope equation was obtained as shown below:

$$\tau_{us} = m \ln(u_a - u_w) + n. \tag{14}$$

However, while the matric suction is equal to zero, the suction strength is not equal to zero. In order to eliminate this problem, the matric suction, $(u_a - u_w)$ and suction strength, τ_{us} , may be transformed to $[(u_a - u_w) + P_{at}]$ and $[\tau_{us} + P_{at}]$. The suction strength equation becomes:

$$\tau_{us} + P_{at} = [m \ln(u_a - u_w) + P_{at}] + n. \tag{15}$$

The parameters m and n in Eq. 15 are the test parameters. When $(u_a - u_w)$ is equal to zero, τ_{us} will be equal to zero. If $(u_a - u_w) = 0$ and $\tau_{us} = 0$ in Eq. 15, n is found as follows:

$$n = -m \ln(P_{at}) \tag{16}$$

substituting Eq. 16 into Eq. 15, and rearranging, produces:

$$\tau_{us} = m \ln \left[\frac{(u_a - u_w) + P_{at}}{P_{at}} \right] \tag{17}$$

in which P_{at} is atmospheric pressure. The parameter m , which reflects the soil properties, is determined by

Table 3 Test results of residual clay

Matric suction, $(u_a - u_w)$ (kPa)	50	100	200	400
Total cohesion, c_{total} (kPa)	35.24	46.72	69.56	98.32

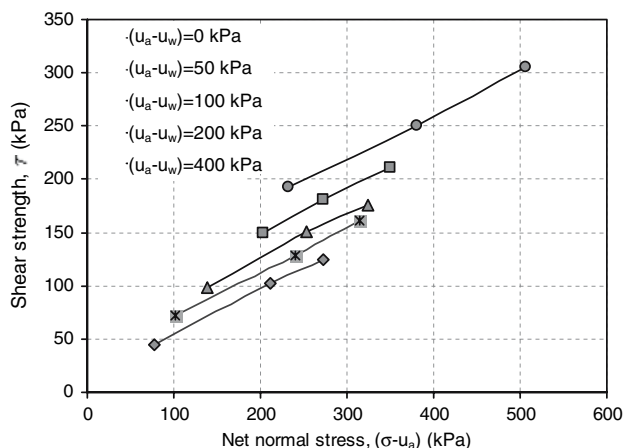


Fig. 8 Shear strength versus net normal stress for different matric suctions

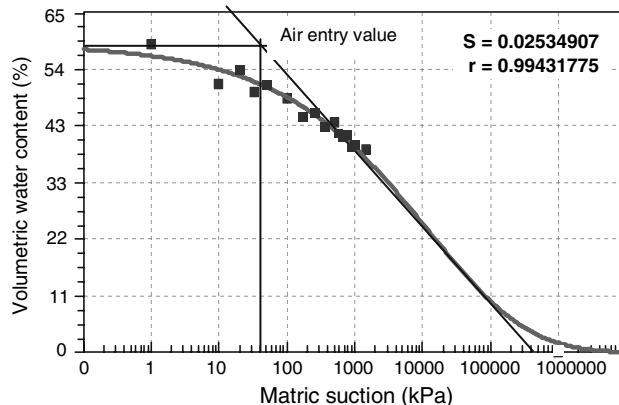


Fig. 9 SWCC of the soil used in this study

regressive analysis of test data of unsaturated soils. By using a basic theoretical requirement of unsaturated soil mechanics, the parameter m can be approximately calculated. When differentiation of the suction strength equation with respect to the matric suction is evaluated at $(u_a - u_w) = (u_a - u_w)_b$ [$(u_a - u_w)_b$ is air entry value of soil], $\tan\phi^b$ is equal to $\tan\phi'$ (Rassam and Cook 2002). For this purpose, differentiating Eq. 17 with respect to $(u_a - u_w)$, we obtain:

$$\frac{d \tau_{us}}{d(u_a - u_w)} = \frac{m}{((u_a - u_w) + P_{at})} = \tan \phi^b. \tag{18}$$

When Eq. 18 is evaluated at $(u_a - u_w) = (u_a - u_w)_b$, $\tan\phi^b$ will be equal to $\tan\phi'$. The parameter m is found as follows:

$$m = \tan \phi' ((u_a - u_w)_b + P_{at}) \tag{19}$$

replacing Eq. 19 in Eq. 17, we obtain:

$$\tau_{us} = \tan\phi' [(u_a - u_w)_b + P_{at}] \ln \left[\frac{(u_a - u_w) + P_{at}}{P_{at}} \right]. \tag{20}$$

Some researchers assume that the soils are saturated for suction values ranging between zero and the air entry value, $0 < (u_a - u_w) < (u_a - u_w)_b$, and, hence, the slope of the suction strength envelope is constant and equal to $\tan\phi'$ within this range (Vanapalli et al. 1996; Rassam and Cook 2002). Although this is clearly not the case for the shear strength relationship given by Eq. 20, it gives acceptable results that are close to the linear line between the zero and air entry value of matric suction. On the other hand, in most cases of engineering applications, the shear strength corresponding to matric suction varying between the air entry and residual value of soils is of interest (Fredlund and Rahardjo 1993; Rassam and Cook 2002). The advantage of Eq. 20, compared to the proposed methods, is that it only requires knowing a single value from SWCC, while the other proposed methods herein need more than one. Consequently, the proposed equation seems to be simple and practical since it requires an air entry value and effective angle of shearing resistance of soil.

Equation 20 reflects the contribution of the matric suction to the shear strength. This contribution might be thought to be part of the total cohesion of unsaturated soils. So, total cohesion can be written as shown below:

$$c_{total} = c' + \tan\phi' ((u_a - u_w)_b + P_{at}) \ln \left[\frac{(u_a - u_w) + P_{at}}{P_{at}} \right]. \tag{21}$$

The logarithmic model of suction strength was used to estimate the suction strength of Diyarbakir residual clays.

The variation of total cohesion with the matric suction predicted from Eq. 21 is graphically compared with those measured values in Fig. 10. The continuous line in the figure represents the predicted values, and the labels are the measured values. One can see that there is good comparison between those predicted from Eq. 21 and the measured values of total cohesion. In Fig. 11, the suction strength values calculated from Eqs. 20, 5, 6 and 7 were also compared with experimental results. As far as Fig. 11 is concerned, Eq. 20 is in better agreement with experimental results in comparison with the Eqs. 6 and 7. The Eq. 5 suggested by Vanapalli gives good prediction up to matric suction of 200 kPa, but it significantly diverges from the test result after the 200 kPa. Consequently, the logarithmic model of suction strength proposed in this paper appears to be a reliable and advantageous method. It requires one parameter, which is the air entry value of soil from the SWCC, to estimate the suction strength. The air entry value can be easily determined by performing the pressure plate test or similar test. The entire stage of the pressure plate test needn't be performed to determine the air entry value of an unsaturated soil, since its value generally has a magnitude lower than 250 kPa. Thus, it is adequate to perform this test up to 250 kPa. Considering that the unsaturated soil tests are fairly difficult, time consuming and expensive, it can be concluded that the use of a logarithmic model is a reasonable method in the determination of the suction strength.

Comparison of several suction equations with known experimental data

In order to further examine the application of the logarithmic model (Eq. 20) and other suction strength

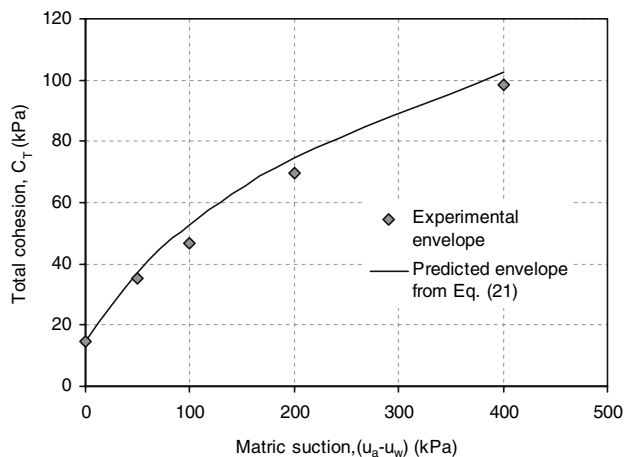


Fig. 10 Total cohesion versus matric suction

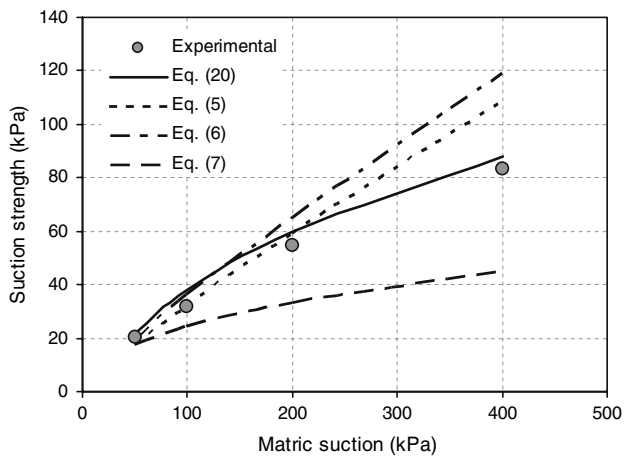


Fig. 11 Comparison of measured and predicted suction strength values for Diyarbakir residual soil

Table 4 Strength parameters

Reference	Soil type	ϕ' (°)	$(u_a - u_w)_b$ (kPa)
Miao et al. (2002)	Nanyang expansive soil	21.3	25
Gan et al. (1988)	Glacial till	25.5	35
Vanapalli et al. (1996)	Glacial till (compacted at optimum condition)	23.0	32

equations given in earlier sections of this paper, experimental results of three soils from the literature were analyzed. The shear strength parameters and SWCC data of the soils used for comparison were summarized in Table 4.

Figure 12 shows comparisons between the predicted and measured suction strength values for Nanyang expansive

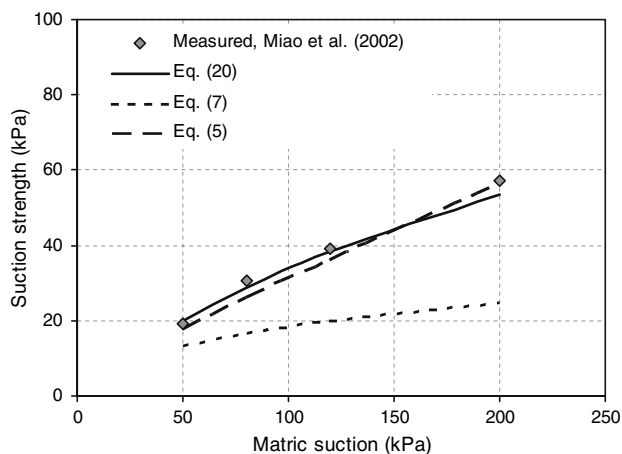


Fig. 12 Comparison of measured and predicted suction strength values for Nanyang expansive soil

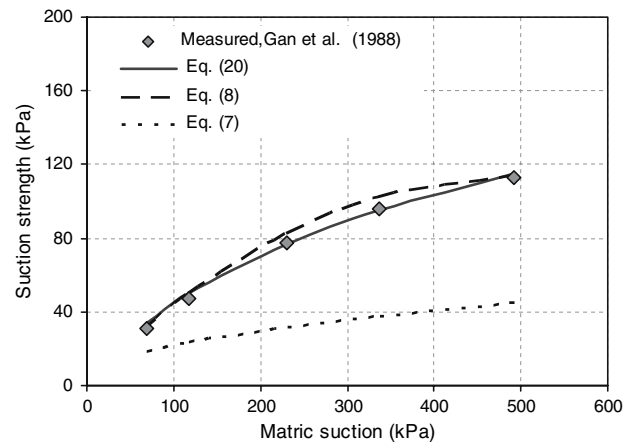


Fig. 13 Comparison of measured and predicted suction strength values for glacial till

soil presented by Miao et al. (2002). There is good comparison between the experimental results and the predicted values using Eqs. 20 and 5. On the other hand, Eq. 7 gives fairly different results from the experimental ones.

Gan et al. (1988) performed a series of unsaturated direct shear tests to determination of shear strength parameters of glacial till. Their tests results and predicted results are shown in Fig. 13. As can be seen from this figure, the predicted envelopes using Eqs. 20 and 8 are very close to the experimental envelope.

In Fig. 14, experimental results of glacial till compacted at optimum conditions reported by Vanapalli et al. (1996) are compared with the predicted suction strength values. It can be seen that suction strength envelopes obtained from Eqs. 20 and 5 are nicely in good agreement with the experimental envelope. Equation 7 does not provide good comparison.

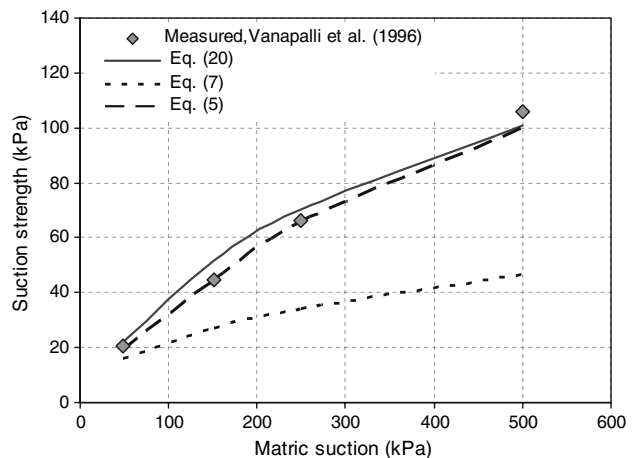


Fig. 14 Comparison of measured and predicted suction strength values for glacial till compacted at optimum conditions

Conclusion

The conventional triaxial test apparatus was modified to determine the unsaturated shear strength parameters for a residual clayey soil from Diyarbakir in Turkey. The apparatus is fully controlled by means of a computer. In the experiments, undisturbed residual soil specimens having a 50-mm diameter and a 100-mm height were used, and matric suction was controlled by using axis translation technique. It was observed that the contribution of matric suction to the shear strength varies non-linearly with matric suction. Due to the difficulties and limitations in the experimentally determination of the shear strength of the unsaturated soils, it appears to be a more reasonable way to use the empirical equations. For this reason, the logarithmic model of suction strength was proposed in this paper. The validity of the logarithmic model proposed was tested for experimental results, and a number of suction strength data are available in the literature. The calculation of the logarithmic model satisfactorily agrees with the experimental data of suction strength obtained both in the present and published studies. It would appear that the use of the logarithmic model is quite adequate for practical engineering purposes and most suitable for clayey soils. The main advantage of the logarithmic model is that it only requires knowing the air entry value of soil from SWCC.

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