

# Growth of continental plateaus by channel injection: models designed to address constraints and thermomechanical consistency

S. MEDVEDEV<sup>1,2,3</sup> & C. BEAUMONT<sup>1</sup>

<sup>1</sup>*Department of Oceanography, Dalhousie University, Halifax, NS, B3H4J1 Canada*

<sup>2</sup>*Fachrichtung Geologie, Freie Universität Berlin, Malteserstrasse 74-100, Berlin, 12249 Germany*

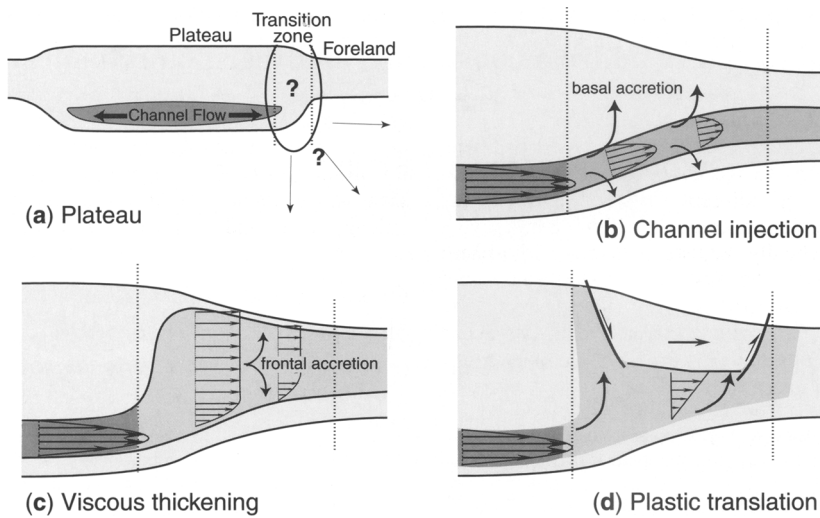
<sup>3</sup>*Present address: Physics of Geological Processes, University of Oslo, PO Box 1048 Blindern, 0316 Oslo, Norway (e-mail: sergeim@fys.uui.no)*

**Abstract:** Weak, possibly partially molten, middle crust may exist and deform by channel flow beneath continental plateaus, thereby significantly influencing their dynamics. The role of channel flows in the transition zone between the plateau and the foreland is, however, unclear. We develop successively more complete approximate models for the channel injection (CI) mode in which differential pressure pumps channel material from beneath the plateau into the transitional crust, which thickens it and widens the plateau. The motivation is to improve our understanding of the controls on the growth of continental plateaus and the interactions in the transition zone, and to gain more insight into the results of more complex numerical models. In model CI-1, a channel with constant viscosity and thickness exists in the transitional crust and the pumped material accretes/freezes above and below the channel. Although results compare favourably with the geometry of some natural examples, this model is incomplete because the connection between the transition zone and the plateau is not considered. Model CI-2 includes a decrease in channel viscosity when the channel depth exceeds a critical value,  $D^*$ , a proxy for onset of melt weakening or low viscosities at high temperatures. The model completes the connection to the plateau, but relies on the arbitrary choice of  $D^*$ . Model CI-3 is more physically based, and considers the channel viscosity and thickness to depend on temperature, calculated by an associated thermal model that includes radioactive self-heating, and advection of heat by channel material. This model demonstrates self-consistent plateau widening if the channel viscosity decreases at the critical temperature,  $T^*$ . Acceptable comparisons with the topography of Tibet are achieved with transition zone viscosities that decrease from  $10^{19}$ – $10^{22}$  Pa s to subplateau values of  $10^{18}$ – $10^{19}$  Pa s, with  $T^*$  of 700–750°C. Additional analyses and tests are used to determine the range of parameter values for which CI models are both internally consistent and compatible with observations. Additional modes of deformation in the transition zone, viscous thickening (VT) and plastic translation (PT), may also be important.

High temperatures and partial melting may lead to low-viscosity channel flows in the mid- to lower crust beneath continental plateaus (Fig. 1a). Following earlier work demonstrating the feasibility of such channel flows (e.g. Bird 1991; Kaufman & Royden 1994), several recent studies indicate that these flows lead to decoupling of deformation with focused flow in the channel and overlying crust that remains relatively stable (e.g. Royden 1996; Beaumont *et al.* 2001, 2004; Shen *et al.* 2001; Vanderhaeghe *et al.* 2003). These models also predict that plateaus grow outward by tectonic thickening and accretion of adjacent crust in the transition zone between the plateau and the surrounding foreland (Fig. 1c) and that channel flows may subsequently convert the transition zone to become part of the plateau. In contrast, Clark & Royden (2000) proposed that crust in the transition zone may thicken by direct injection of channel material (Fig. 1b) and showed

this style to be compatible with the crustal thickening of the eastern margin of the Tibetan plateau. In this proposed channel injection (CI) mode no crustal shortening in the transition zone is required, as suggested by the limited shortening observed from the eastern margin of the Tibetan plateau.

A primary motivation for further development of channel injection models, as described here, is to improve our understanding of the first-order controls on the growth of continental plateaus and the interactions that occur within the transition zone. The second motivation is to gain more insight into predictions of numerical models which produce realistic-looking results but are difficult to interpret in terms of the basic physical controls (Beaumont *et al.* 2001, 2004; Jamieson *et al.* 2004). We investigate some of the basic factors controlling channel flows, and ask whether they contribute to the outward growth of plateaus as in the channel injection mode.



**Fig. 1.** Diagram showing possible models for the growth of a continental plateau and its relationship to the adjacent transition zone. (a) There is strong evidence for weak mid-/lower crust below plateaus and channel flow may also occur at depth. Less is known about the relationship between the plateau and the transition zone. Three processes may occur in this zone adjacent to a neutral growing plateau: (b) channel injection (CI); (c) crustal thickening by accretion, in this case considered to be viscous thickening (VT); (d) outward flow of unstable brittle upper and mid-crust, termed plastic translation (PT). Darker grey areas represent zones of high strain rate.

The concepts that form the basis for this work (Fig. 1) rely on the existence of weak crust beneath plateaus. The existence of weak subplateau crust is also consistent with the 'flat' plateau topography in that weak mid- to lower crust implies that the viscosity here is too low to create sufficient deviatoric stress to maintain long-wavelength components of the topography, and therefore plateaus flatten like pancake batter. In contrast, steep topographic gradients in the plateau-flanking transition zones (Fig. 1a) imply stronger, higher effective viscosity crust, and/or a change in boundary conditions that allows these zones to laterally support (like bookends) the plateau against its tendency to flow outward. In the limit, very weak subplateau crust will undergo outward channel flow driven by the pressure gradient between the plateau and the foreland (Fig. 1a), but can this limited differential pressure inject channel material into the transition zone?

Possible mechanisms for crustal thickening in the transition zone can be modelled as channel injection (Fig. 1b; Clark & Royden 2000), viscous tectonic thickening (Fig. 1c; Royden 1996; Shen *et al.* 2001), and plastic translation (Fig. 1d; Lehner 2000; Gemmer *et al.* 2004), in which the upper brittle, frictional-plastic crust decouples and rides outward above the ductile substratum. Channel injection can thicken the transitional crust without surface shortening, but viscous thickening and

plastic translation require tectonic contraction, and in the last instance extension as well.

Two end-member settings also exist for plateau growth: the 'actively convergent margins' of the plateau for which contraction is imposed at the lithospheric scale by large-scale plate motions (e.g. the north and south margins of the Tibetan plateau), and the plate-tectonically 'neutral margins', which are less affected by plate-scale process but may be tectonically contractional owing to the gravitationally driven outward flows of the plateau material (e.g. the eastern margin of Tibetan Plateau). Clearly, channel injection can, in principle, be the sole mechanism that thickens the crust in the transition zone of neutral margins, but there must be a component of tectonic contraction of the crust in actively convergent margins. This distinction separates plateau transition zones into two classes: the actively convergent type modelled in Beaumont *et al.* (2001, 2004) and Vanderhaeghe *et al.* (2003); and the neutral type modelled by Clark & Royden (2000). The three-dimensional models (Royden 1996; Shen *et al.* 2001) can address both types of boundary.

The two fundamentally different types of boundary raises a 'chicken-and-egg' issue in regard to the role of channel injection in crustal thickening and the growth of plateaus. Can injection of channel material occur directly into normal continental

crust, as modelled by Clark & Royden (2000), or must there be a precursory phase of tectonic thickening followed by thermal incubation to develop the hot, weak conditions under which channel injection will operate efficiently?

In this paper, successively more complete approximations to the evolution of a continental plateau and the adjacent transition zone by the channel injection mechanism within crust above a stable isostatically adjusting mantle lithosphere are examined. The first two models, CI-1 and CI-2, largely ignore the thermal aspects of the problem. In CI-3 an associated thermal model is introduced which allows both the thickness and the temperature/viscosity of the channel to be predicted. The last part of the paper addresses the question of mechanical and thermal consistency by restricting the acceptable range of model parameter values to those thermal conditions for which the mechanics of channel injection will both operate efficiently and give results compatible with observations. This analysis also includes a comparison of the CI model with simplified models of viscous thickening (Fig. 1c) and plastic translation (Fig. 1d) of the crust.

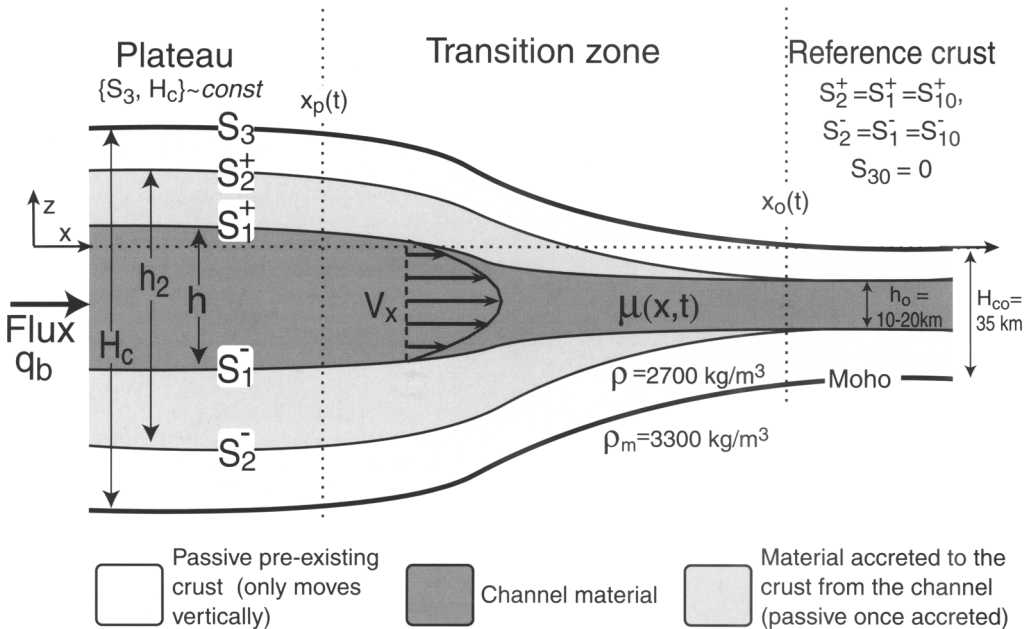
The possible range of parameter values is large, therefore we restrict the models to those that produce similar geometries to those of Beaumont *et al.* (2004) for the transition zone (plateau high approximately 5 km, transition zone 100–150 km) at a similar time during their evolution (*c.* 20 million years). The ranges of consistent rheological and boundary conditions are then compared among models and with natural examples.

### Channel injection theory

The simple shear thin sheet approximation (Medvedev & Podladchikov 1999; Lehner 2000) and mass balance (Equation 1) describe the flow in a linearly viscous, locally isostatically balanced, crustal channel

$$\begin{aligned} \frac{\partial S_2^+}{\partial t} + \Phi \frac{\partial q}{\partial x} &= 0 \\ \frac{\partial h_2}{\partial t} + \frac{\partial q}{\partial x} &= 0 \end{aligned} \quad (1)$$

The geometry and boundary conditions (Fig. 2) are as follows:  $S_2^+$  is the upper boundary of the



**Fig. 2.** General channel injection (CI) model. Three crustal zones are considered: reference undeformed foreland crust; the transition zone (where crust thickens by accretion of injected material from/within the channel bounded by  $S_2^\pm$ ); the plateau (characterized by nearly flat topography,  $S_3 = S_{3p} = \text{const}$ , and constant thickness crust,  $H_c = H_{cp}$ ). Crust has uniform density  $\rho$ . The boundary condition is the flux of material from the plateau into the channel,  $q_b$ . Material flow in the transition-zone channel comprises a Poiseuille flow driven by the transitional-zone topographic gradient. The underlying mantle lithosphere, density  $\rho_m$ , is stable and moves vertically in response to local isostatic adjustment of the system.

channel material,  $\Phi = (\rho_m - \rho_c)/\rho_m$  is the isostatic amplification factor and  $\rho_m$  and  $\rho_c$  are respectively the densities of mantle and crust. The material flux  $q(x)$  through the channel, thickness  $h(x)$ , assuming lithostatic pressure and uniform density of the crust, is

$$q = \int_{S_1^-}^{S_1^+} V_x dz = -\frac{\rho g h^3}{12\mu} \frac{\partial S_3}{\partial x} \quad (2)$$

where  $S_1^+$  and  $S_1^-$  are the boundaries of the channel,  $\mu$  is the viscosity of the channel material, and  $S_3$  is the upper surface of the crust. The variation in  $S_3$  induces lateral pressure gradients which drive the Poiseuille flow. The horizontal velocity inside the channel is

$$V_x(z) = -\frac{\rho g (z - S_1^-)(S_1^+ - z)}{2\mu} \frac{\partial S_3}{\partial x} \quad (3)$$

The general model of the channel injection (Fig. 2) assumes three zones in the crust and an underlying stable mantle lithosphere that undergoes local isostatic adjustment. In the crust, material between  $S_2^+$  and  $S_3$  and between  $S_2^-$  and  $S_M$  is pre-existing crust above and below the channel, and the material in this zone moves only vertically and acts as a passive load. Material between  $S_1^+$  and  $S_1^-$  is the channel material actively involved in the current flow. The material between  $S_1^+$  and  $S_2^+$ , and between  $S_1^-$  and  $S_2^-$  is material that accreted to the crust from the channel, and is passive once accreted. Equations 1 and 2 can be solved for a range of assumptions concerning the channel thickness  $h(x)$ , and the channel viscosity  $\mu(x)$ , as described in the next sections.

### Channel injection model CI-1: transition zone with uniform property channel

This model, in which  $h$  and  $\mu$  are constants, was used by Clark & Royden (2000) to calculate channel flow in the plateau-bounding transition zone and the results were applied to the eastern flanks of the Tibetan Plateau. In addition to the uniform thickness and viscosity they assumed that material accretes (freezes) to the channel walls as soon as it crosses  $S_1^\pm$ , thereby thickening the region between  $S_1^+$  and  $S_2^+$  and  $S_1^-$  and  $S_2^-$ . This material then acts as part of the passive overburden by applying its weight to create a pressure gradient along the channel. In this model the geometry of the channel does not change with time ( $h = \text{const}$ ), even though the velocity field and topography do evolve. Equations 1 and 2 can be combined for

this case, and using  $S_3 - S_2^\pm = \text{const}$  (i.e. constant thickness of the initial crust above the channel) gives:

$$\frac{\partial S_3}{\partial t} = \frac{\partial S_2^+}{\partial t} = \frac{\Phi \rho g h^3}{12} \frac{\partial}{\partial x} \left( \frac{1}{\mu} \frac{\partial S_3}{\partial x} \right) \quad (4)$$

Using the assumptions of Clark & Royden (2000), that the channel has constant viscosity and the flux into the model is constant ( $\mu = \text{const}$ ,  $q_b = \text{const}$ ), we solved Equation 4 analytically using the Laplace method (Özişik 1980, chapter 7) for the evolution of the upper surface:

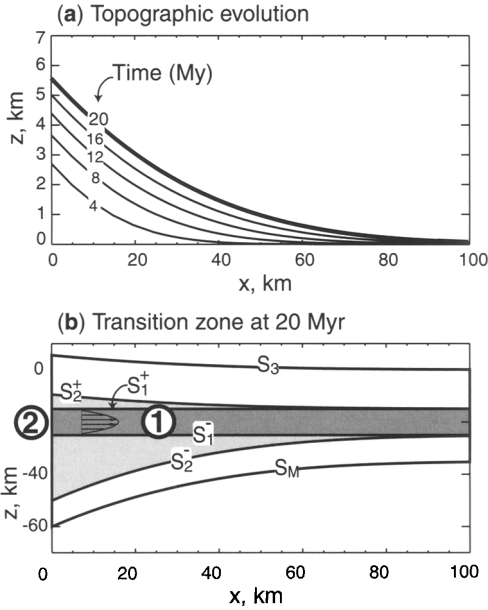
$$S_3(x, t) = q_b \left[ \sqrt{\frac{\alpha t}{\pi}} \exp\left(-\frac{x^2}{\alpha t}\right) - x \operatorname{erfc}\left(\frac{x}{\sqrt{\alpha t}}\right) \right] \quad (5)$$

where  $S_3(x, t = 0) = 0$  is the initial position of upper surface,  $\alpha = (\Phi \rho g h^3)/3\mu$ ,  $\operatorname{erfc}$  is the complementary error function, and  $t$  is time.

Based on this approach, Clark & Royden (2000) estimated the distribution of viscosity in putative crustal channels under the north, east and SE margins of the Tibetan Plateau using the current topography of those margins ( $S_3$ ). However, as they acknowledge, the results are critically dependent on parameters that cannot be accurately estimated (i.e. the values of  $h$  and  $q_b$ ) and whether  $h$ ,  $q_b$  and  $\mu$  are actually constant. If uncertainties in these parameters are taken into account (for example,  $h$  ranges from 5 to 20 km), the resulting estimates for channel viscosity,  $\mu$ , can vary by orders of magnitude even when  $q_b$  is known because the control parameter  $\alpha$  scales as  $h^3/\mu$ . Therefore, an uncertainty in  $h$  of a factor of slightly more than 2 causes an order of magnitude uncertainty in  $\mu$ .

Figure 3 illustrates a typical result for model CI-1. Assuming constant flux of material into the system,  $q_b = 10^{-6} \text{ m}^3 \text{ s}^{-1}$  per m along-strike (which is equivalent to an average rate of  $3.3 \text{ mm a}^{-1}$  in a 10 km thick channel), results in elevating the plateau side of the transition zone to 5.5 km in 20 million years. Although the results in Figure 3a were obtained for the special case of 10 km thick channel with  $\mu = 3 \times 10^{20} \text{ Pa s}$ , exactly the same results can be achieved for 20 km thick channel with  $\mu_{20} = 2.4 \times 10^{21} \text{ Pa s}$ , where the subscript indicates the assumed thickness of the channel in kilometres which yields this estimate of the viscosity.

While the results of Clark & Royden (2000) compare well with the topography of the chosen margins of the Tibetan Plateau, there is a need to address additional questions that cannot be



**Fig. 3.** Results for channel injection model CI-1. The special case considered here includes:  $\mu = 3 \times 10^{20}$  Pa s,  $h = 10$  km. Flux of material into the system,  $q_b = 10^{-6} \text{ m}^3 \text{ s}^{-1}$  per m along strike (average rate is  $3.3 \text{ mm a}^{-1}$  in 10 km channel), results in the plateau margin being elevated to 5.5 km in 20 million years. Results are the same for constant  $h^3/\mu$  (e.g.  $\mu_{15} = 10^{21}$  Pa s (for  $h = 15$  km) or  $\mu_{20} = 2.4 \times 10^{21}$  Pa s (for  $h = 20$  km). Note: (1) results do not depend on the vertical position of the channel within the crust; (2) model does not address connection with plateau.

answered by channel injection model CI-1. For example, how is the flux  $q_b$  determined dynamically and would the available differential pressure drive this particular flux? How can this model of the transition zone be matched to the geometry of the plateau itself, and how are the overall dynamics of these two regions related?

#### Channel injection model CI-2: constant thickness, variable viscosity channel

The approach of Clark & Royden (2000) can be combined with that of Royden (1996) which used a depth-dependent model of viscosity. Based on the assumption that temperature increases with depth in the crust and that viscosity correspondingly decreases, we propose a model in which the channel develops at the base of the upper crust (Fig. 4). In this model there is a step decrease in the channel viscosity when the centre of the channel, depth  $d$ , reaches a critical depth  $D^*$  (Fig. 4). This decrease

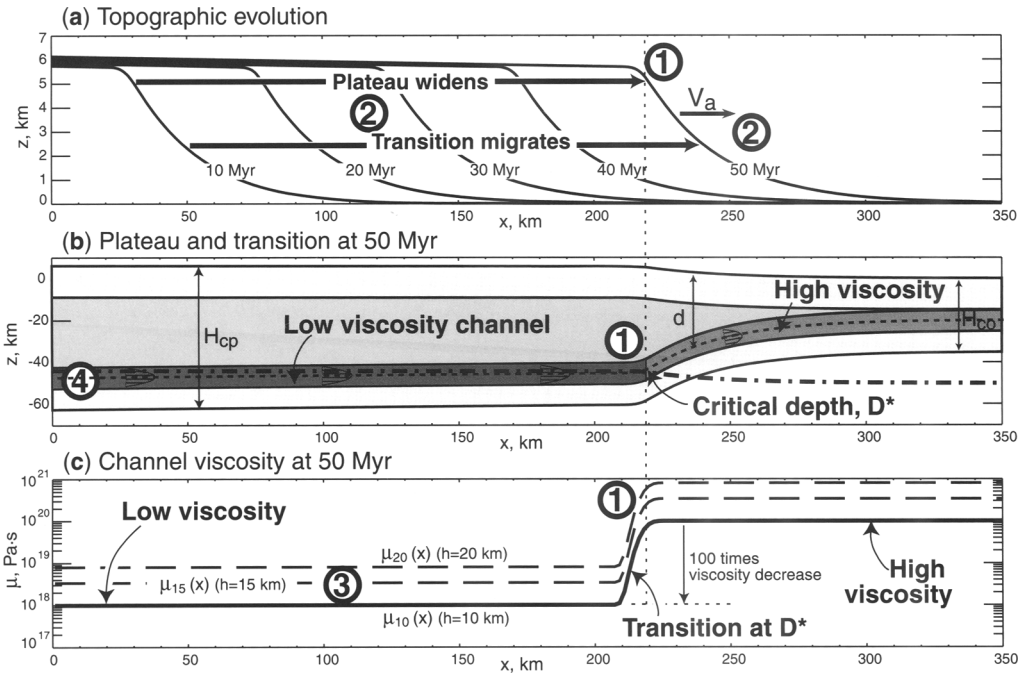
can be interpreted to be the consequence of partial melting. Thus, viscosity in the channel is  $\mu = \mu(x, t, D^*)$ , and the evolution of the system is described by the same Equation 4 as in the model CI-1. This model can address both the development of the transition zone ( $d < D^*$ ) and the adjacent plateau ( $d > D^*$ ).

The boundary flux  $q_b$  and viscosity  $\mu(d)$  were chosen in a similar way to model CI-1 (Fig. 3) and to Clark & Royden (2000), so that the edge of the plateau grows to a height of 5.5 km in 20 million years, and also so that geometry of the transition zone is similar to that in the previous section (Fig. 3). For example, the result shown in Figure 4 displays a plateau-like geometry of the orogen if the viscosity of the channel decreases by a factor of 100 under the plateau ( $\mu(d > D^*) = \mu(d < D^*)/100$ , Fig. 4b,c). The absolute value of viscosity can be estimated, as in the model CI-1, within the uncertainty of the control parameter  $h^3/\mu$  (Fig. 4c). The dynamic link between the plateau and its adjoining transition zone results in more conservative estimates in general of the viscosity under the plateau ( $\mu \approx 10^{18} - 10^{19}$  Pa s) than estimates presented in Clark & Royden (2000), where estimates based on the model CI-1 gave  $\mu \approx 10^{16}$  Pa s for equivalent  $h$  values.

Although model CI-2 is very simple, it expresses a bulk behaviour similar to full two-dimensional (2D) numerical models of plateau evolution (Vanderhaeghe *et al.* 2003). Once the plateau is established, it grows laterally and the geometry of the transitional zones remains self-similar (Fig. 4a) in the same manner noted by Vanderhaeghe *et al.* (2003). A simple kinematic analysis allows the rate at which the plateau edge advances,  $V_a$ , and the corresponding rate of plateau widening to be estimated. For the boundary conditions implemented here (i.e. the edge of plateau grows from 0 up to 5.5 km in 20 million years) the edge of the plateau advances by the width of the transition zone in the same timescale ( $V_a \approx 5 \text{ mm a}^{-1}$ ). Mass balance relates  $V_a$  to the boundary flux by  $q_b = V_a \Delta H_c$ , and thus

$$V_a = q_b / \Delta H_c \approx \text{const} \quad (6)$$

where  $\Delta H_c = H_{cp} - H_{c0} \approx \text{const}$  is the difference in the crustal thickness between the foreland and the plateau. Although the lateral variations of the plateau crustal thickness are small, the plateau crust must thicken internally (Fig. 4a) to drive the subplateau channel, because the rate of flow in the channel is proportional to the local slope of topography. This requirement, however, is an artefact of the thin-sheet approximation used here to describe the system. Beaumont *et al.* (2004) describes channel



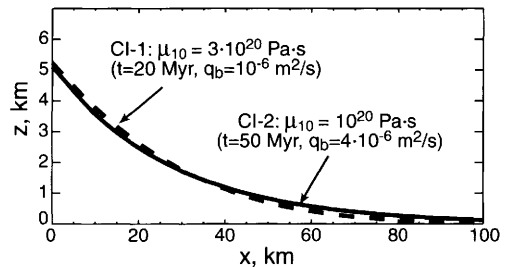
**Fig. 4.** Results for channel injection model CI-2. Special case considered here: thickness of the channel  $h = 10$  km;  $\mu = 10^{20}$  Pa s if the depth of the centre of the channel,  $d(x, t) < D^* = 50$  km, and  $\mu = 10^{18}$  Pa s otherwise. Flux of material into the system,  $q_b = 4 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$  per m along strike (average rate is  $1.5 \text{ cm a}^{-1}$  in 10 km channel) results in plateau margin elevated to 5.5 km in 20 million years. Note: (1) decrease in viscosity under transitional crust leads to plateau-type topography; (2) self-similar widening of the plateau is characterized by the plateau advancing velocity  $V_a$  and by kinematic waveform of evolution of upper surface  $S_3 = S_3(x + V_a t)$ ; (3) results are the same for constant  $h^3/\mu$ ; (4) channel flux varies spatially: under the plateau  $q \approx q_b = \text{const}$ , and in the transition zone flux decreases.

flow under a nearly flat plateau using full 2D numerical calculations for which the driving force for the channel flow is not the local, but global changes in topography. We therefore simplify model CI-2 and assume constant thickness plateau crust,  $H_{cp} = \text{const}$ , and, correspondingly,  $\Delta H_c = \text{const}$  in subsequent analysis.

The rate of plateau edge advance,  $V_a$ , is not a material velocity, nor is it associated with the motion of any physical part of the system. The self-similar evolution of the plateau is similar to the propagation of a kinematic wave such that evolution of the topography in the transition zone can be expressed as  $S_3(x, t) = S_3(x - \Delta t V_a, t - \Delta t)$  for any positive  $x$ ,  $t$  and  $\Delta t (< t)$ . We use this property and assume that other model parameters (for example, temperature in the thermal model; see Appendix) also evolve in the form of a laterally propagating kinematic wave.

Similar geometries and dynamics of growth (5.5 km over 20 million years) can be obtained for models CI-1 and CI-2 (Fig. 5) only if the viscosity of the channel under the transition zone in model

CI-2 is approximately three times smaller than in model CI-1. Model CI-2 also requires a four times higher boundary flux into the system. The



**Fig. 5.** Comparison of the final geometry of channel injection model CI-1 (dashed line, see details on Fig. 3) and transitional zone predicted by the channel injection model CI-2 designed to achieve the same geometry (solid line, see details on Fig. 4). Similar geometry is, however, obtained for different viscosities and flux into the system because the model CI-2 accounts for the connection to plateau. See text for details.

discrepancy is caused by an oversimplification in model CI-1, which does not account for the additional flux required to build the advancing plateau in addition to the flux into the translating transition zone. Building the plateau margin comprises two processes: first, the development of the transition zone as in model CI-1; and second, the conversion of the margin to plateau, which is the process that requires the extra material flux. In this regard model CI-2 is more complete than CI-1.

The kinematic connection between the transition zone and the plateau edge in model CI-2 also results in a time-dependent flux into the transition zone at a fixed location, even though the boundary flux  $q_b$  is a constant. Under the plateau, where the thickening rate is negligible, the flux in the channel  $q \approx q_b = \text{const}$ . The channel material flux then decreases across the transition zone to zero at the toe. Thus, the model CI-1 assumption of a constant flux into the transition zone cannot be combined with the plateau advancing with a constant rate  $V_a$ .

### *Channel injection model CI-3: temperature-dependent channel properties*

While the resulting geometry of model CI-2 looks realistic (Fig. 4), we question whether the physical foundation of the model is correct. Is depth a good proxy for temperature in determining channel properties? Would a better model based on thermal evolution of the crust result in different channel properties and flow regimes?

Vanderhaeghe *et al.* (2003) compared depth- and temperature-dependent representations of crustal viscosity in models of plateau evolution. Although the topographic shape of the resulting plateau can be similar for both rheologies, the internal structure of the plateau crust is substantially different. In the model in which viscosity decreases with depth (see also Royden 1996; Shen *et al.* 2001), a low-viscosity weak layer of constant thickness develops below the plateau and is reflected by the flat topography of the plateau. In contrast, the temperature-dependent viscosity model of the crust results in large (up to 20 km) variations in thickness of the low-viscosity weak layer and, consequently, inversely correlated variations in thickness of the overlying upper, strong and cold layer (Vanderhaeghe *et al.* 2003). The geometry of the weak crustal layer determines the structure of the channel flow in the crust; therefore, we can expect different interactions between the overburden and the channel for the depth- and temperature-dependent models.

Here we combine model CI-2 with an approximate thermal model (Appendix) to analyse evolution of the plateau and the transition zone when the viscosity and geometry of the channel is temperature-dependent. We assume that instead of a

critical depth,  $D^*$ , the channel viscosity decreases when the channel temperature reaches a critical value,  $T^*$ , corresponding to the onset of significant melt weakening. The channel thickness, initially  $h_0$ , increases to include the mid- and upper crust where  $T > T^*$ , but not the lower crust which is assumed to be at granulite facies and not subject to melting. This model approximates radioactive self-heating, partial melting, weakening of crustal material, and a consequently increasing region of actively deforming low-viscosity crust, with a critical temperature  $T^* = 750^\circ\text{C}$ , chosen to represent the effect of muscovite dehydration melting (Johannes & Holtz 1996).

The associated thermal model (Appendix), which calculates the one-dimensional vertical thermal evolution, and the self-similar widening of the plateau (Fig. 4), is used to determine the horizontal distribution of temperature  $T(x, z, t) = T(z, t - x/V_a)$ . To make the results of models CI-2 and CI-3 comparable, we chose the thermal parameters of the model (Table 1) so that the mid-channel temperature also reaches the critical value after 20 million years of transition-zone thickening.

The results (Fig. 6) look similar to those of model CI-2 (Fig. 4), but in this case the channel becomes thicker as the plateau crust self-heats and approaches a steady thermal state. The thermal dependence of the channel geometry provides an independent self-consistent measure for thickness of the channel, whereas in models CI-1 and CI-2 the choice of channel thickness is arbitrary. Comparison of the model topographies (Fig. 7) shows that a similar geometry to that of CI-2 is obtained for CI-3 with a smaller decrease in viscosity because the channel thickness increases from its initial value,  $h_0$ , in CI-3, but remains constant in CI-2 (compare Figs 4 and 6).

### **Thermal and rheological consistency**

The previous section presented model CI-3, which combines mechanical and thermal models. These are separable into two parts. The next step is to analyse the relationship between these two parts for internal consistency, and consistency with natural systems. We first discuss the correlation between melt weakening, its consequences for the boundary between the transition zone and plateau, and then examine the effect of temperature variations along the channel in the transition zone. Lastly, we ask whether the viscosity of channel material implied by the model results is consistent with laboratory measurements of quartz-controlled rheologies.

**Table 1.** *Parameters of the reference model*

Variable	Definition	Value
<i>Geometry</i>		
$H_c$	Thickness of the model crust	
$H_{c0}$	Initial crustal thickness	35 km
$H_{cp}$	Crustal thickness at the edge of plateau	65 km
$h_2$	Thickness of deformable part of the crust (Fig. 2)	10 → 40 km
$h$	Thickness of the crustal channel in the CI model (Fig. 2)	$h_0 = 10$ km
$d$	Depth of the middle of the channel	$d_0 = 20$ km
$h_3$	Thickness of the lower, undeformable crust (Fig. 2)	10 km
$S_{3p}$	Uplift of topography at the plateau edge ( $S_{30} = 0$ , Fig. 2)	5.5 km
<i>Mechanical parameters</i>		
$t^*$	Thickening time	$2 \times 10^7$ years
$g$	Acceleration due to gravity	$9.8 \text{ m s}^{-2}$
$\rho$	Density of the crust	$2700 \text{ kg m}^{-3}$
$\rho_m$	Density of the lithospheric mantle	$3300 \text{ kg m}^{-3}$
$\Phi$	Isostatic amplification factor ( $1 - \rho/\rho_m$ )	0.18
$\mu$	Viscosity of the channel material	Variable (Pa s)
<i>Thermal parameters</i>		
$T(x, z, t)$	Temperature (Eqn A1)	(°C)
$T_{top}$	Temperature on the top surface (boundary condition for Eqn A1)	0°C
$T^*$	Critical temperature (Fig. 6)	750°C
$q_m$	Mantle heat flux (boundary condition for Eqn A1)	$20 \text{ mW m}^{-2}$
$C_p$	Heat capacity (Eqn A1)	$750 \text{ J kg}^{-1} \text{ K}^{-1}$
$K$	Thermal conductivity (Eqn A1)	$2 \text{ W m}^{-1} \text{ K}^{-1}$
$\kappa$	Thermal diffusivity ( $= K/\rho C_p$ , Eqn A1)	$10^{-6} \text{ m}^2 \text{ s}^{-1}$
$A_1$	Volumic rate of heat production in the upper crust (initially 20 km thick, Fig. A2)	$1.8 \mu\text{W m}^{-3}$
$A_2$	Volumic rate of heat production in the lower crust (initially between 20 and 35 km deep in the crust, Fig. A2)	$0.75 \mu\text{W m}^{-3}$

### *Criteria for initial and transient consistency of thermal and mechanical models*

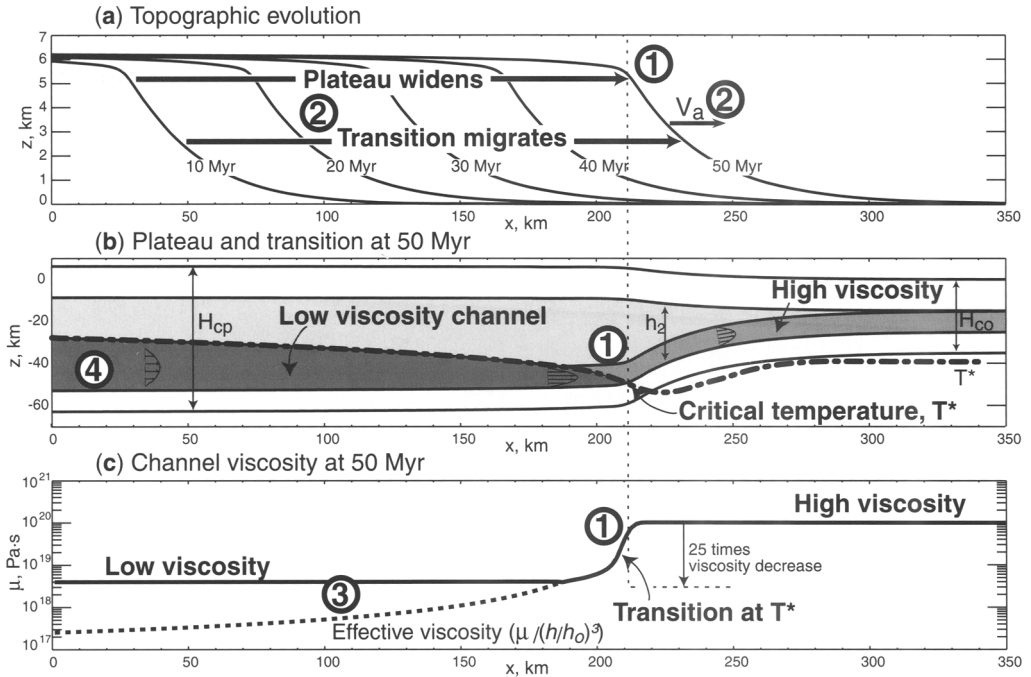
Three criteria can be used to analyse consistency between the mechanical and thermal models.

1. *Initial conditions:* if channel flow is the mechanism that tectonically thickens the crust, temperatures in the crust must be high enough, and associated viscosity must be low enough, to allow viscous channel flow to occur under the initial ambient conditions.
2. *Temperature change:* the temperature change in each crustal column, while it is in the transition zone, should be such that the final temperature meets the requirements for channel viscosity to decrease by melt weakening, so that the transition to plateau topography will occur.
3. *Timescale of temperature change:* the timescale for increase in temperature and onset of melt weakening of the transition zone must agree with that of the mechanical thickening

model, so that the mechanical and thermal plateau transitions are compatible.

The first criterion addresses the initial thermal state of undeformed crust. Our reference model (Table 1, Fig. 6) with  $h_0 = 10$  km is consistent because the temperature at the top of the channel is 400°C (Fig. A2), and the threshold for viscous creep in quartz-controlled crust is approximately 300°C. Moreover, any equivalent foreland crust is consistent with this temperature requirement if the radioactive heat production is  $0.75 \mu\text{W m}^{-3}$  or higher. However, a requirement for a thicker channel, e.g.  $h_0 = 15$  km, is marginally too cold (290°C at the top of the channel) unless the thermal parameters are changed. Increasing radioactive heat production would, for example, require  $A = 5.4 \mu\text{W m}^{-3}$  for the corresponding channel to be 20 km thick.

The second criterion determines whether thickening the crust results in efficient heating. Crustal thickening results in transient heating, but this occurs with a characteristic timescale often longer

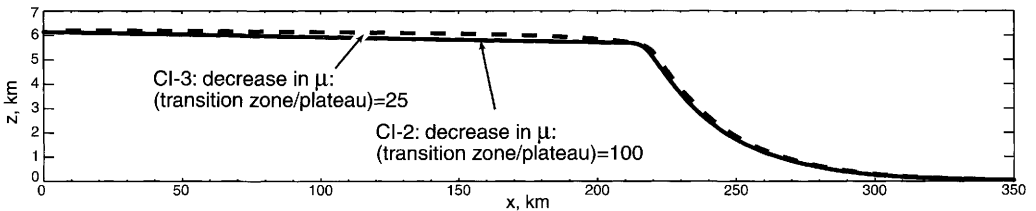


**Fig. 6.** Results for channel injection model CI-3. Special case considered here:  $\mu = 10^{20}$  Pa s if temperature in the channel is below critical  $T < T^* = 750^\circ\text{C}$ , and  $\mu = 4 \times 10^{18}$  Pa s if  $T > T^*$ . Channel thickness is initially  $h_0$ . It then increases to include mid- and upper crust where  $T > T^*$ . Other parameters are the same as in Figure 4. Note: (1) decrease in viscosity under transitional crust leads to plateau-like geometry; (2) self-similar widening of the plateau is characterized by the plateau advancing at velocity  $V_a$  and by kinematic waveform of evolution of upper surface  $S_3 = S_3(x + V_a t)$ ; (3) control parameter for the model is  $h^3/\mu$  (demonstrated by the effective viscosity, dotted line on (c), but here  $h$  and  $\mu$  are determined independently by the thermal state); (4) channel flux varies spatially: under the plateau  $q \approx q_b = \text{const}$  and in the transition zone flux decreases.

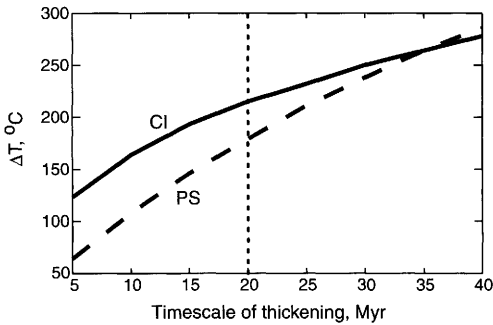
than the timescale for mechanical thickening. Comparison of the increase of the temperature at the base of the crust versus the timescale for crustal thickening (Fig. 8) shows that fast mechanical thickening (timescale  $< 10$  million years) results in only moderate increases in temperature so that both mechanisms, the channel injection (CI) and pure shear (PS), (see Appendix) are ineffective. For

thickening timescales that are very long ( $> 40$  million years), both mechanisms predict significant heating. At intermediate timescales, which are common in natural orogens, CI is seen to be a more efficient mechanism for crustal heating than PS.

The longer-term differences are a consequence of the relative contributions from radioactivity. Pure shear thickens the crust uniformly producing a



**Fig. 7.** Comparison of the final geometries of channel injection models CI-3 (dashed line, see details on Fig. 6) and CI-2 (solid line, see details on Fig. 4). Similar geometry is obtained, but for a different viscosity decrease under the plateau owing to thicker channel in the model CI-3. See text for details.



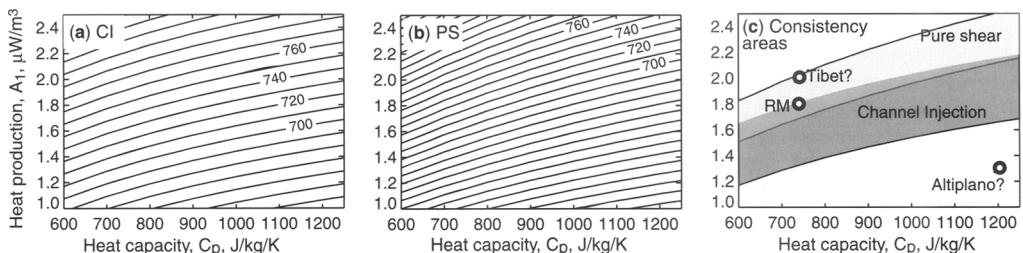
**Fig. 8.** Comparison of temperature rise after thickening the crust by 30 km at a constant rate for either the channel injection mode (solid line) or pure shear mode of thickening (dashed line). The thermal state depends on the time during which thickening occurs, but CI heats crust faster than PS. Note that although the initial distribution of radioactive heat sources are equal for two models, the PS model results in a thicker, highly radioactive crustal layer (up to 44 km at the edge of plateau) than model CI-3 (which thickens the upper, high-radioactivity crust up to 22 km). See details of the thermal model in Appendix. Dotted vertical line indicates duration of thickening for the reference model.

44 km thick high-radioactivity layer (this layer is only 22 km thick in model CI), but the crust self-heats slowly, redistributing heat conductively. Channel injection heating is more efficient for thickening timescales less than 40 million years because CI injects hot material from lower in the crust and channel flow advects heat from hot crust beneath the plateau (Appendix).

The third consistency criterion concerns the timing of the thermal evolution in the transition zone and requires that the base of the crust be close to the melting temperature at the time the transition zone transforms into a plateau. This transformation is interpreted to be a consequence of the decrease in viscosity owing to melt weakening.

Figure 9a shows the temperature predicted at the depth of the middle of the channel for model CI-3 after 20 million years of crustal thickening as a function of the radioactivity of the upper crust,  $A_1$ , and thermal capacity,  $C_p$  (other parameter values are as given Table 1). High values of  $A_1$  and low values of  $C_p$  produce temperatures compatible with the onset of melting. Equivalent results for the PS model (Fig. 9b) are similar but the crust is colder for the same parameter values. Rather than analysing for consistent models we reject inconsistent ones on the basis that melting most likely occurs in the temperature range 700–750°C (Johannes & Holtz 1996). Models that do not achieve this temperature range in 20 million years, and models that achieve these temperatures too quickly, were rejected leaving those with parameters in the range shown by the two shaded bands (Fig. 9c) for the CI-3 and PS models. Similar plots can be made for other combinations of parameters and duration of thickening.

Particular cases can also be analysed for consistency. For example, parameter values used by Beaumont *et al.* (2001, 2004) for Himalyan–Tibetan models are inconsistent with thickening by CI-3 but consistent with PS (Fig. 9c). This result also agrees with their numerical model results which showed PS thickening in the transition zone and channel flows only penetrating this region when there is significant erosion. The parameters used by Babeyko *et al.* (2002) to model the Altiplano are incompatible with both modes of thickening on the 20 million years timescale and would require a minimum 35 million years interval of thickening for CI to operate and 45 million years for PS. If the Altiplano crust is expanding outward and thickening the transition zone it must have an additional source of heat not considered in our CI and PS models. This conclusion was also reached by Babeyko *et al.* (2002) who applied a high mantle heat flux,  $60 \mu\text{W m}^{-2}$ , in order to achieve the necessary conditions.

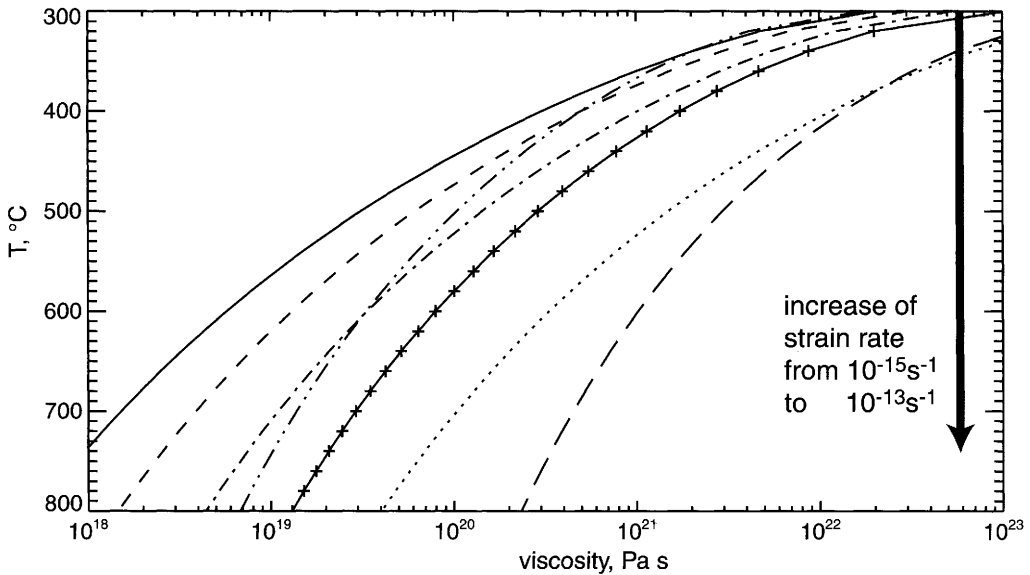


**Fig. 9.** Contour plots of final temperature in middle of the transitional channel for channel injection (a) and pure shear (b) models. (c) Areas of thermal parameters of the crust (radioactivity of the upper crust,  $A_1$  (initial thickness 20 km) and heat capacity,  $C_p$ ) that can result in a consistency between thermal and mechanical models (grey). Other parameters are the same as in reference model (Table 1). Circles illustrate thermal parameters of reference model (RM), and thermal parameters used to model Tibetan Plateau (Beaumont *et al.* 2004) and Andes Altiplano (Babeyko *et al.* 2002).

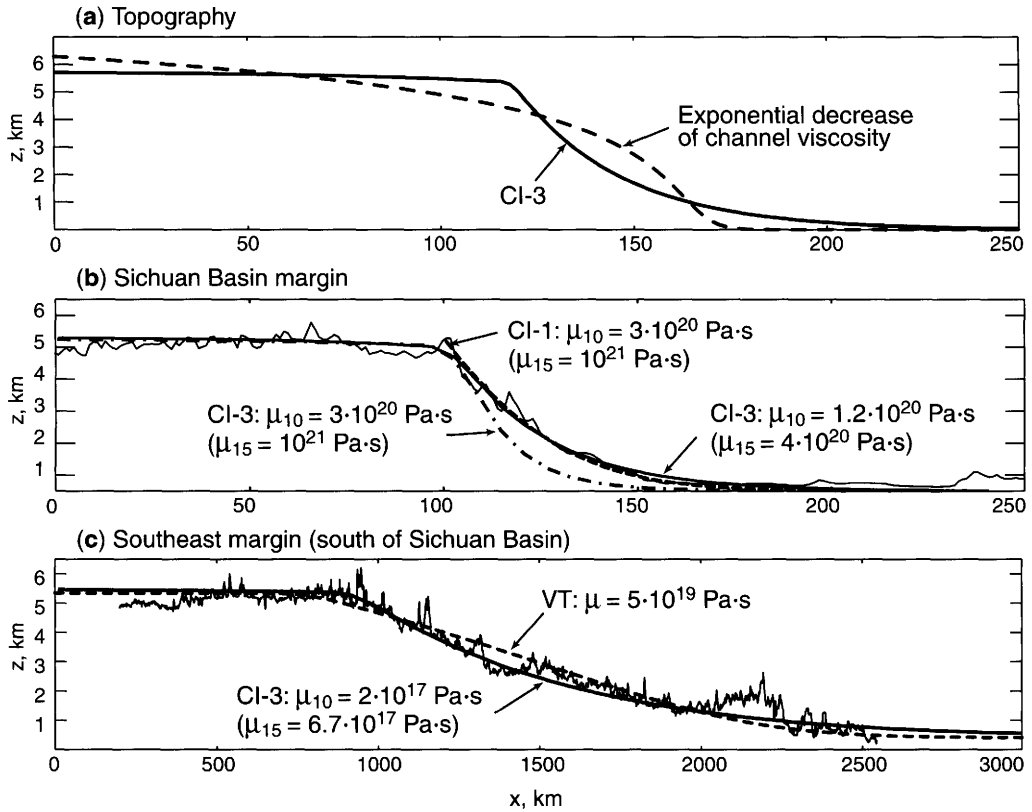
### Criteria for rheological consistency of the channel below the transition zone

In this section we investigate under what circumstances channel injection is consistent with laboratory measurements of the rheological properties of quartz-rich rocks. Natural plateaus usually exhibit an abrupt boundary between the transition zone and the plateau, which we have interpreted to reflect a significant step decrease in channel viscosity related to the onset of partial melting (Fig. 6). An alternative explanation is that the viscosity variation of quartz-rich crust with temperature and strain rate is sufficient to explain the abrupt plateau-edge transition without melt weakening. The large uncertainty in the effective viscosity of quartz-rich rocks is seen from Figure 10, which shows laboratory data (table 3 in Gleason & Tullis 1995) plotted as effective viscosity versus temperature. The plot differs from the usual ones of this type in that the assumed strain rate is not constant but varies from  $10^{-15} \text{ s}^{-1}$  to  $10^{-13} \text{ s}^{-1}$ , to reflect the anticipated variation in strain rate between the foreland end of the channel, where temperatures may be as low as 300–400°C, and the plateau end of the transition zone, where temperatures may reach 700–800°C, and the strain rates are larger.

The results show that the extrapolated effective viscosity estimates are approximately  $10^{21} \text{ Pa s}$  with a factor of 3 uncertainty at 400°C, and  $10^{19} \text{ Pa s}$  at 750°C, but with a much larger factor of uncertainty of approximately 10. If we eliminate the two lowest viscosity estimates based on the reliability of the laboratory measurements as discussed by Gleason & Tullis (1995), we predict that for a temperature range of 400–750°C, calculated for the channel beneath the transition zone, associated viscosities should correspondingly vary from  $10^{21} \text{ Pa s}$  to  $10^{19} \text{ Pa s}$ . Figure 11a shows the plateau-edge geometry for this choice of viscosity variation compared with that predicted by the CI-3 model with constant viscosity in the transitional channel and an abrupt decrease in the viscosity at the plateau edge. Despite the admittedly large uncertainties, it can be seen that the geometry based on extrapolated laboratory data, and not including melt weakening, fails to predict the abrupt plateau-edge geometry. In addition, this model predicts that topography increases steeply at the toe, much more steeply than in the CI-3 model (Fig. 11a). This steepness is because channel flow is very inefficient at viscosities higher than  $10^{20} \text{ Pa s}$ , which suggests that normal crustal thickening at the foreland end of the transition zone is not rheologically consistent



**Fig. 10.** Dependence of effective viscosity of quartzite on temperature and strain rate. Here we approximate thermal and strain regime inside the channel in the transitional zone. Foreland end of channel is characterized by low temperature (300–400°C) and low strain rate ( $\leq 10^{15} \text{ s}^{-1}$ ). Plateau end of transitional channel has high temperature (700–800°C) and high strain rate (up to  $2 \times 10^{13} \text{ s}^{-1}$  for the reference model). Plot shows variations of effective viscosity of more than two orders of magnitude over evolution of the transitional channel. Lines show effective viscosity versus temperature, for the assumed  $10^{-13}$  to  $10^{-15} \text{ s}^{-1}$  range of strain rates, compared from several sets of experimental laboratory creep data on the deformation of quartz-rich rocks. Data compiled by Gleason & Tullis (1995) and shown in their table 3.



**Fig. 11.** Comparison of model CI-3 results with experimental and natural examples. (a) Result of model CI-3 (constant viscosity along channel and melt weakening at the edge of plateau, solid line) differs significantly from model with an exponential decrease of channel viscosity during evolution of the orogen (dashed line). The latter model (see text) is based on laboratory results presented in Figure 10. (b) Comparison of model results with topographic profile across Sichuan Basin margin of Tibetan Plateau (topographic data modified from Clark & Royden 2000, fig. 4c). Results for CI-3 ( $\mu = 1.2 \times 10^{20}$  Pa s for channel thickness 10 km, solid line) and CI-1 ( $\mu = 3 \times 10^{20}$  for  $h = 10$  km, dashed line; see also fig. 4c in Clark & Royden 2000) almost coincide in the transitional zone (between  $x = 100$  and 200 km) and match well the natural topography, but differ by a factor of  $c. 3$ , the value of the estimated viscosity. Model CI-1 does not account for the development of the plateau. For comparison, the dash-dot line shows result for model CI-3 using the value of the viscosity of the channel material in the transition zone estimated by CI-1. (c) Comparison of models results with topographic profile across the SE margin of the Tibetan Plateau (topographic data modified from Clark & Royden 2000, fig. 4a). The result obtained by model CI-3 (solid line) compares well with the natural example, but the predicted viscosity of the channel material is unreasonably low unless the lower crust was exceptionally hot at the time of channel injection. Viscous thickening (VT, dashed line) produces a more reasonable estimate of the average viscosity of the entire orogenic crust.

with the channel injection model, and that an alternative mechanism needs to be considered.

A similar question of rheological consistency arises in regard to the estimates by Clark & Royden (2000). Whereas estimated viscosities for the channel material under the margins with short transitional zones (e.g. Sichuan Basin margin of Tibetan Plateau, Fig. 11b) may be consistent with the laboratory data, the estimate of channel viscosities of  $10^{18}$  Pa s for a 2000 km wide region of the SE Tibetan Plateau margin south of the Sichuan

Basin (and even lower viscosity predicted by the more complete CI-3 model, Fig. 11c) is questionable. Such low viscosities require temperatures of approximately  $800^\circ\text{C}$ , even for the lower effective viscosity extrapolations of the laboratory data (Fig. 10). The results imply that the channel injection explanation for crustal thickening only applies if the crust in this region was already anomalously hot prior to channel injection, which raises the related question of whether it was already anomalously thick.

## Behaviour of the upper crust

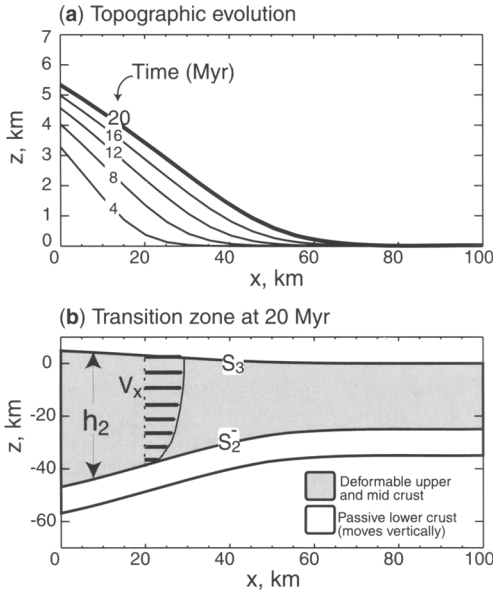
We complete the analysis with a preliminary description of other mechanisms, viscous thickening (VT) and plastic translation (PT) (Fig. 1c and d), which may also contribute to deformation and crustal thickening of the transition zone but differ from channel injection.

### Viscous thickening model

The CI models assume that crust above the channel is much stronger than the channel material and therefore does not deform during injection. However, if this is not correct the deformation can instead be evenly distributed in the transitional crust. Simple models of this type treat the deformation as viscous, are based on the simple shear thin-sheet approximation (e.g. Buck & Sokoutis 1994; Royden 1996; Medvedev 2002), and assume that the upper surface of the crust is deformable and stress-free. The remaining boundary conditions are the same as those for the CI model.

The VT model is of this type (Fig. 12) and the mass balance gives

$$\frac{\partial S_3}{\partial t} + \Phi \frac{\partial q_v}{\partial x} = 0 \quad (7)$$



**Fig. 12.** Results for viscous thickening model. Special case considered here has  $\mu = 1.5 \times 10^{22}$  Pa s. Flux of material into the system,  $q_b = 10^{-6} \text{ m}^3 \text{ s}^{-1}$  per m along-strike (as for Fig. 3) results in plateau margin elevated to 5.5 km in 20 million years. The geometry of the transitional zone is similar to one by CI-3 (Fig. 7), but rheological parameters of these two models differ significantly.

which is similar to Equations 1 and 2, except the corresponding flux

$$q_v = -\frac{\rho g h_2^3}{3\mu} \frac{\partial S_3}{\partial x} \quad (8)$$

results from integration of the velocity profile, which is

$$V_x(z) = -\frac{\rho g (z - S_1^-)(h_2 + S_1^+ - z)}{2\mu} \frac{\partial S_3}{\partial x} \quad (9)$$

for a uniformly viscous crust and assuming lithostatic pressure.

The resulting system of equations is similar to those for the CI model (Equations 1–3), except for the denominator 3, instead of 12, in the expressions for the fluxes. This difference occurs because the upper surface in the VT model is free to deform. Note also that the VT equations are non-linear ( $h_2$  is a linear function of  $S_3$ ) and cannot be solved analytically. The model result (Fig. 12) is shown for the viscosity  $1.5 \times 10^{22}$  Pa s which produces the same transition-zone geometry as models CI-2 and CI-3 with channel material viscosity of  $10^{20}$  Pa s.

The difference of two orders of magnitude in viscosity required to produce the same topography for the transition zone demonstrates the sensitivity of the behaviour to the mode of deformation. This result implies that for natural examples the type of deformation must be known, in addition to the geometry, before the viscosity can be accurately estimated. An example is the very wide (c. 2000 km) transition zone SE of the Sichuan basin for which Clark & Royden (2000) estimated a viscosity of  $10^{18}$  Pa s based on the CI model (model CI-3 predicts an even lower value for viscosity; Fig. 11c). The corresponding estimate from the VT model is  $5 \times 10^{19}$  Pa s, which may be more realistic if contraction at the surface cannot be ruled out.

### Stability of the upper crust

The last mode of deformation in the transition zone considers the plastic (brittle) failure and translation (PT) of the crust above the channel injection zone. Here we estimate the stability of the upper crust, which allows us to distinguish when the CI and PT modes will operate. Lehner (2000) considered the stability of frictional overburden coupled to a viscous substratum and used the thin-sheet approach for the channel flow (Equation 1) and the small angle approximation for the overburden. Gemmer *et al.* (2004) presented a simplified version of the theory for systems with constant-thickness channels and they also derive an expression for the velocity when the crust is unstable. They showed that their results

are in good agreement with full 2D numerical calculations.

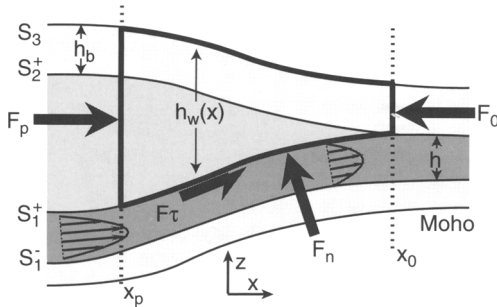
Consider the forces acting on a block of the crust above the channel in the transition zone (Fig. 13, area surrounded by bold line). The horizontal balance of forces is:

$$F_p - F_0 - F_{nx} + F_\tau = 0 \quad (10)$$

where  $F_p$  and  $F_0$  are the normal forces acting on the block from the edge of plateau and from the foreland respectively,  $F_\tau$  is the traction on the base of the block produced by the Poiseuille flow in the transitional channel, and  $F_{nx}$  is the horizontal component of the buoyancy force. If the forces in Equation 10 cannot be balanced, the block becomes unstable and starts to move. The forces can be estimated in the following ways. The wedge-shaped block, thickness  $h_w(x)$ , has two parts, an upper uniform frictional-plastic layer, thickness  $h_b$ , and a lower approximately triangular-shaped viscous region above the channel and below the plastic layer. Therefore

$$F_p = \int_{S_1^+(x_p)}^{S_3(x_p)} \sigma_{xx} dz = F_{pb} + F_{pv} \quad (11)$$

where  $F_{pb}$  and  $F_{pv}$  are the integrals of the horizontal stress within the plastic and viscous parts, respectively, at  $x = x_p$ . Integration of the stress under the small-angle assumption for the geometry of the block, and assuming that the principal stresses are horizontal and vertical and the pressure is equal to the lithostatic pressure, gives (Gemmer *et al.* 2004):  $F_{pb} > F_p^* = \rho g h_b^2 k/2$  and  $F_{pv} = \rho g (h_{wp}^2 - h_b^2)/2$ ;  $k = (1 - \sin \phi)/(1 + \sin \phi)$  is the coefficient of passive earth pressure (Lehner 2000), where  $\phi$  is



**Fig. 13.** Geometry and forces for simplified analysis of the stability of brittle upper crust of the transition zone (block with bold outline). The brittle block is underlain by a mid-crustal channel and material accreted from the channel. Only forces that contribute to the horizontal balance are indicated.

the internal angle of friction of the overburden, and  $h_{wp} = h_w(x_p)$  is the thickness of the system at the edge of plateau.  $F_p^*$  is the limiting value of  $F_{pb}$  when the system is on the verge of failure, or has failed. Under the equivalent assumptions,  $F_0 < F_0^* = \rho g h_b^2 k^{-1}/2$ .

The horizontal component of the normal stress at the base of the system can be calculated following Medvedev & Podladchikov (1999) by estimating the buoyancy by integrating the pressure along the base of the block

$$\begin{aligned} F_{nx} &= \int_{x_0}^{x_p} \rho g h_w(x) \frac{\partial S_1^+(x)}{\partial x} dz \\ &= \frac{\rho^2 g}{2\rho_m} \int_{x_0}^{x_p} \frac{\partial h_w^2}{\partial x} dz = \frac{\rho^2 g}{2\rho_m} (h_{wp}^2 - h_b^2) \end{aligned} \quad (12)$$

Here we use the constraint that the boundary  $S_1^+$  is parallel to the base of the crust (Fig. 13), and the Airy isostatic condition

$$\frac{\partial S_1^+}{\partial x} = \frac{\rho}{\rho_m} \frac{\partial h_w}{\partial x} \quad (13)$$

where  $\rho_m$  is the density of the lithospheric mantle (Table 1). The traction force from the viscous flow in the channel is determined by integrating the basal drag caused by the channel injection along the length the block (Gemmer *et al.* 2004):

$$\begin{aligned} F_\tau &= \frac{\rho g h}{2} (S_3(x_p) - S_3(x_0)) \\ &= \frac{\rho \Phi g h}{2} (h_{wp} - h_b) \end{aligned} \quad (14)$$

where  $\Phi$  is the isostatic amplification factor (Table 1). Substitution of the force estimates into Equation 10 gives the stability condition for the system:

$$h_b^2(k - k^{-1}) + \Phi(h_{wp}^2 - h_b^2) + \Phi h(h_{wp} - h_b) < 0 \quad (15)$$

Equation 15 can be solved for the critical limiting value of  $\phi$ ,  $\phi^*$  (below which the block becomes unstable) using the geometry (Fig. 6) and properties (Table 1) of our reference model. Here we use  $h_b = 15$  km, based on the depth to the 400°C isotherm calculated using the associated one-dimensional thermal model (Appendix) and shown in Figure A2. Crust below this level will almost certainly be viscous. If  $h_{wp} = 40$  km,  $\phi^* = 21.5^\circ$ . The block will be unstable if  $\phi < \phi^*$ . It is also worth noting that erosion which occurs sufficiently rapidly to reduce the thickness of the frictional-plastic layer will tend to destabilize the system by

reducing  $F_p^*$  and/or  $F_\delta^*$ . This effect is seen in the numerical models (Beaumont *et al.* 2004, fig. 7) and leads to the transition from channel flow beneath stable plateau and transitional crust to unstable outward flow of the upper crust together with the channel. For example, for the current reference model a reduction of the frictional–plastic layer thickness to 10 km would render the block unstable for  $\phi = 21.5^\circ$  and requires an increase to  $\phi^* = 37^\circ$  if the system is to remain stable.

If the system becomes unstable, the overburden fails, the block glides outward with velocity  $V_C$ , and the force balance (Equation 10) becomes

$$F_p^* + F_{pv} - F_0^* - F_{nx} + F_\tau - F_C - F_V = 0 \quad (16)$$

where the forces  $F_{pb}$  and  $F_0$  are replaced by their limiting values because the frictional–plastic overburden has failed.  $F_C = \mu L_C V_C/h$  is an additional force owing to the basal drag of the Couette flow in the viscous channel on the overburden (Gemmer *et al.* 2004), and  $L_C = x_p - x_0$ .  $F_V$  is the additional viscous force that results from the extension of the viscous crust above the channel at the edge of the plateau. This force can be estimated by assuming that the triangular region necks viscously with a horizontal length scale  $L_n$ , which gives

$$F_p = \int_{S_1^+(x_p)}^{S_2^+(x_p)} 2\mu_w \frac{V_C}{L_n} dz = 2\mu_w (h_{wp} - h_b) \frac{V_C}{L_n} \quad (17)$$

assuming that the triangular block has a uniform viscosity  $\mu_w$  in the region that necks.

The following results are obtained assuming  $h_b = 15$  km,  $L_C = 100$  km,  $L_n = 30$  km, and  $\mu_w = 10^{22}$  Pa s, and using other parameter values from Table 1. The maximum velocity  $V_C = 8$  mm a<sup>-1</sup> occurs when  $\phi = 0^\circ$ , and ranges from 6 mm a<sup>-1</sup> ( $\phi = 5^\circ$ ), through 5 mm a<sup>-1</sup> ( $\phi = 9^\circ$ ), to 2.75 mm a<sup>-1</sup> ( $\phi = 15^\circ$ ). These results demonstrate that passive plateau margin transition zones can be close to extensional failure and the outward flow velocities can be significant if the viscosity of the crust above the channel at the plateau margin is approximately  $10^{22}$  Pa s. The outward flow could still be significant on geological timescales even if this viscosity were an order of magnitude higher.

## Conclusions

Simple modelling has been used to improve our understanding of the first-order controls on the growth of continental plateaus and the interactions that occur within the transition zone at the plateau margin. The simplified approach of this study also helps us to gain more insight into the results of

numerical models which produce realistic-looking geometries, but are difficult to interpret in terms of the basic physical controls (Beaumont *et al.* 2001, 2004; Jamieson *et al.* 2004).

*Three channel injection (CI) models.* We developed successively more complete approximations to the evolution of a continental plateau and adjacent transition zone by the CI mode. CI-1, the simplest approximation (based on Clark & Royden 2000), assumes a constant viscosity and thickness channel in which excess material accretes in zones above and below the channel. While the predictions of this model compare well with some natural examples, this model does not consider relations between the plateau and transition zone. CI-2 includes a decrease in channel viscosity, predicted to occur on melt weakening or at high temperatures, when the crustal thickness exceeds a critical value,  $D^*$ . This model completes the connection between plateau and transition zone, but relies on an arbitrarily chosen critical thickness of the crust. CI-3 is more realistic and considers the channel viscosity and thickness to be temperature-dependent. CI-3 includes an associated thermal model which accounts for radioactive self-heating of the transition crust and for heat advection as channel material flows from beneath the plateau. CI-3 demonstrates self-consistent plateau widening if the channel viscosity decreases when its temperature exceeds a critical value  $T^*$ .

The simplicity of the models allows analytical solutions for some cases, and provides direct estimates of the parameters that control channel flow. Acceptable results for the topography of Tibet, for example, are achieved by model CI-3 when channel viscosity is in the range  $10^{18}$ – $10^{19}$  Pa s beneath Tibet and  $10^{19}$ – $10^{22}$  Pa s beneath the adjacent areas, and the critical temperature for onset of partial melting is in the range 700–750°C. These values agree with those determined from fully coupled thermal–mechanical models.

*Consistency of thermal and mechanical models.* The first thermal consistency test requires the initial thermal–viscous structure of the crust to be compatible with crustal thickening by channel injection. The second thermal consistency test asks whether the temperature reaches the melting point, thereby initiating melt weakening, at the appropriate place and time. Comparison of the thermal evolution of model CI-3 with a model in which transitional crust tectonically thickens uniformly (PS) shows that CI is a much more effective mechanism for heating the transitional crust. The third, transient, thermal consistency test assesses whether the development of the melting conditions in the transitional crust coincides with the end of thickening. Premature melt weakening will, for example, result in flattening the transitional-zone

topography before it reaches the level of the plateau, and that is incompatible with lateral growth of the plateau. The requirement for consistency significantly restricts the range of compatible parameter values.

**Rheological consistency.** Rheological consistency compares the model predictions of channel viscosity for the transitional crust with viscosity estimates from laboratory experiments. Modelling of wide ( $1.5\text{--}2 \times 10^3$  km) transitional zones, for example, with the CI-3 model requires channel viscosity to be about  $2\text{--}10 \times 10^{17}$  Pa s, which is incompatible with laboratory data for quartz-based rheologies without ‘melt weakening’.

**Viscous thickening (VT) and plastic translation (PT) of the transitional crust.** The compatibility of the CI models, which assume only vertical passive motion of the upper crust, was compared with possible VT and PT deformation of the upper crust. Predicted CI-3 topography of the transition zone can be reproduced by viscous thickening of the crust, but corresponding VT model estimates of the crustal viscosity are up to two orders of magnitude higher than those for CI. This result indicates that topography alone is not a sufficient basis from which to estimate crustal viscosity. The mode of deformation must also be known. Stability estimates for the failure and outward flow of the plateau margin, PT mode deformation, indicate that neutral plateau transition zones may be close to failure by plastic translation of the crust above a channel.

The research was funded by Natural Sciences and Engineering Research Council grant to C.B. S.M. has also been supported by the Deutsch Forschungsgemeinschaft (Sonderforschungsbereich Grant 267). We thank the reviewers, R. Buck and T. Gerya, and the lead Editor, R. Law, for their constructive comments. M. Clark is thanked for providing data on Tibet margins topography.

## Appendix: the associated thermal model

This appendix develops an approximate one-dimensional (1D) vertical model for temperature in the crust that is thickened by channel injection. It is used in conjunction with channel injection models to investigate how channel flows may develop when viscosity depends on crustal temperature. To emphasize the role of the mode of thickening, we compare the CI mode with the model in which the upper and middle crust thickened by pure shear (PS) mode.

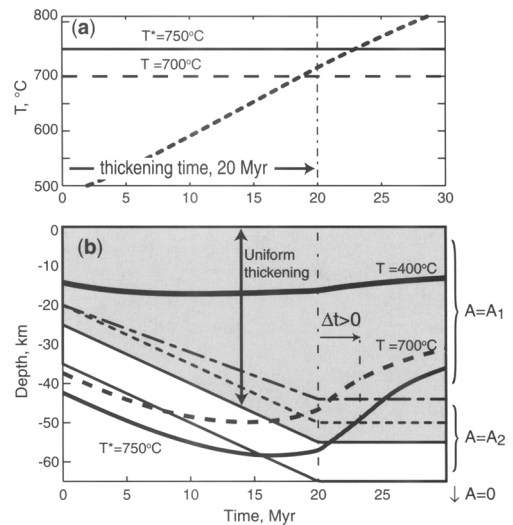
Vanderhaeghe *et al.* (2003) demonstrated that the 2D cross-sectional thermal evolution of plateau crust can in some circumstances be explained approximately by analysis of a reduced vertical 1D thermal problem. We use the same approach here. Based on the result that vertical temperature variations are much larger than horizontal ones (as

determined from the first and second spatial derivatives from 2D calculations), term 6 of the 2D thermal equation

$$\frac{\partial T}{\partial t} = \frac{A}{\rho C_p} - V_z \frac{\partial T}{\partial z} + \kappa \frac{\partial^2 T}{\partial z^2} - V_x \frac{\partial T}{\partial x} + \kappa \frac{\partial^2 T}{\partial x^2} \quad (\text{A1})$$

may be neglected. This assumption is also in agreement with the thin-sheet character of our mechanical model, where horizontal variations are assumed to be small. In Equation A1,  $T$  is the temperature,  $t$  is time,  $A$  is the rate of radioactive self-heating,  $C_p$  is the specific heat at constant pressure,  $V_z$  and  $V_x$  are vertical and horizontal components of velocity,  $\kappa = K/(\rho C_p)$  is the thermal diffusivity, and the numerical values of the parameters are given in Table 1.

**Pure shear (PS) model.** A specific example of pure shear crustal thickening is solved. This model is then used as a reference for comparison with the CI models. We assume that upper and middle crust is thickened at a uniform rate by a factor of 2.2 in 20 million years resulting in a total crustal thickness of 65 km, at which time the crust is assumed to become stable plateau crust (Fig. A1). In addition to the simplification noted above,



**Fig. A1.** Thermal evolution of pure shear model. (a) Evolution of temperature at base of the middle crust (5 km above lower crust, dotted line in b). (b) General view of evolution of a vertical Lagrangian crustal column. Dark grey indicates upper and middle crust that thickens uniformly, light grey is lower, undeformable crust. The dash-dot line separates crust into high-radioactivity ( $A = A_1$ ) and low-radioactivity ( $A = A_1$ ) parts. The underlying mantle lithosphere (the depth of the model is 90–100 km) has zero radioactivity. The isotherms ( $T^* = 750$  °C, solid line;  $T = 700$  °C, dashed line) indicate compatibility of the thermal and mechanical models.

in this model we also neglect horizontal advection (term 5, Equation A1) on the basis that this term is much smaller than vertical advection (term 3, Equation A1) because both velocity components are small, but the vertical temperature gradient is larger. The simplified equation is solved numerically and Figure A1 shows the result for the reference model. Note that the upper, highly radioactive part of the crust thickens significantly from an initial 20 km to 44 km.

*Channel injection model CI-3.* To apply the thermal model to channel injection in the CI-3 model we use the self-similar nature of the lateral growth of the plateau (Figs 4 and 6). We assume that the temperature field also evolves in a self-similar manner, that is,  $T(x, z, t) = T(x + V_a \Delta t, z, t + \Delta t)$ . This implies that during steady-state plateau growth, the vertical thermal structure migrates laterally with the same velocity as the plateau edge,  $V_a$  (Equation 6). Furthermore, in CI-3 the horizontal advection of heat cannot be neglected because  $V_x$  in the channel may be large. The self-similarity of the temperature profile gives

$$\frac{\partial T}{\partial x} = -\frac{1}{V_a} \frac{\partial T}{\partial t} \approx -\frac{1}{V_a} \left( \frac{A}{\rho C_p} - V_z \frac{\partial T}{\partial z} + \kappa \frac{\partial^2 T}{\partial z^2} \right) \quad (\text{A2})$$

where the first equality results from

$$\begin{aligned} \frac{\partial T}{\partial x} &= \lim_{\Delta t \rightarrow 0} \frac{T(x, z, t + \Delta t) - T(x, z, t)}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{T(x, z, t + \Delta t) - T(x + V_a \Delta t, z, t + \Delta t)}{\Delta t} \\ &= -V_a \frac{\partial T}{\partial x} \end{aligned} \quad (\text{A3})$$

The approximate equality in Equation A2 is the zeroth-order approximation of the horizontal temperature gradient in the system. Combining Equations A1 and A2 gives the higher-order approximation for the thermal evolution of crust thickened by channel injection:

$$\frac{\partial T}{\partial t} = (1 + \theta_c) \left( \frac{A}{\rho C_p} - V_z \frac{\partial T}{\partial z} + \kappa \frac{\partial^2 T}{\partial z^2} \right) \quad (\text{A4})$$

Here,  $\theta_c$  is the channel heat advection coefficient. It is zero outside of the channel and  $\theta_c = V_x/V_a$  inside the channel. It can be seen that the horizontal channel flux accelerates the thermal evolution in the vicinity of the channel by a factor  $(1 + \theta_c)$ . The distribution of  $\theta_c$  can be found from the mass balance. First, Equation 1 and self-similarity of geometry of the transition zone give

$$\frac{\partial q_p}{\partial x} = -\frac{\partial h_2}{\partial t} = V_a \frac{\partial h_2}{\partial x} \quad (\text{A5})$$

The requirement that  $q_p = 0$  in the undeformed foreland allows integration of Equation A5:

$$q_p = V_a(h_2 - h_0) \quad (\text{A6})$$

where  $h_0$  is the initial thickness of the channel. In addition, the depth-dependence of the horizontal velocity Equation 3 can be expressed via flux in the channel,  $q_p$ , using Equation 2:

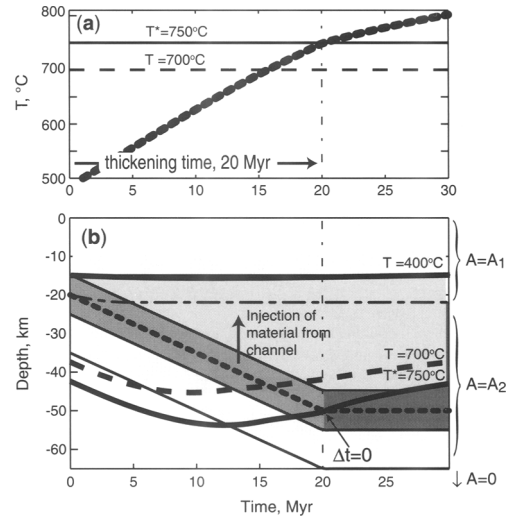
$$V_x(z) = \frac{6q_p}{h^3} (z - S_1^-)(S_1^+ - z) \quad (\text{A7})$$

Combining Equations A6 and A7 we obtain channel heat advection coefficient inside the channel of thickness  $h$ :

$$\theta_c = \frac{V_x}{V_a} = \frac{6(h_2 - h_0)}{h^3} (z - S_1^-)(S_1^+ - z) \quad (\text{A8})$$

We solve Equation A1 (for the pure-shear model) and Equation A4 (for the CI-3 model) numerically using a Lagrangian approach and a finite-difference approximation. The 1D mesh consists of 200–300 grid points with variable density of the mesh (more dense in the upper and mid-crust). The bottom boundary condition (heat flux,  $q_m$ ) is set initially at the depth of 95 km.

To illustrate the results we consider two model properties: the temperature evolution for a point located 5 km above the base of the upper crust (Fig. A2a), and the time when the temperature at this location reaches the critical value (Fig. A2a,b). The model behaviour will be consistent with the channel injection behaviour and melt



**Fig. A2.** Thermal evolution of channel injection (CI) model. (a) Evolution of the mid-channel temperature. (b) General view of thermal and thickening evolution of a vertical Lagrangian crustal column. Levels of grey (from lighter to darker) indicate: upper and low undecomposed crust; material accreted from the channel; high-viscosity channel in the transition crust; low-viscosity channel beneath the plateau. See details of design in Figure A1. Note that highly radioactive portion of the crust thickens much less than in pure shear model.

weakening to form a plateau if, at 20 million years, the lower part of the upper crust reaches a high enough temperature, *c.* 750°C ( $\Delta t = 0$ , meaning the temperature is reached at the specified 20 million year time). Significant lead ( $\Delta t < 0$ ) or lags ( $\Delta t > 0$ ) in the time at which this condition is achieved implies the thermal evolution is not consistent with the mechanical model which assumes that the transition to a plateau occurs at 20 million years. The same consistency conditions can be applied to the plateau model in which the transition zone thickens by pure shear mode (Fig. A1).

We consider two types of thermal consistency. The CI-3 model requires an exact match between the mechanical and thermal models ( $T^* = 750^\circ\text{C}$ ,  $\Delta t = 0$ ). However, in the consistency analysis our uncertainty in the critical temperature is acknowledged, and we consider a range, 700–750°C.

## References

- BABEYKO, A. YU., SOBOLEV, S. V., TRUMBULL, R. B., ONCKEN, O. & LAVIER, L. L. 2002. Numerical models of crustal scale convection and partial melting beneath the Altiplano-Puna plateau. *Earth Planetary Science Letters*, **199**, 373–388.
- BEAUMONT, C., JAMIESON, R. A., NGUYEN, M. H. & LEE, B. 2001. Himalayan tectonics explained by extrusion of a low-viscosity channel coupled to focused surface denudation. *Nature*, **414**, 738–742.
- BEAUMONT, C., JAMIESON, R. A., NGUYEN, M. H. & MEDVEDEV, S. 2004. Crustal channel flows: 1. Numerical models with applications to the tectonics of the Himalayan-Tibetan Orogen. *Journal of Geophysical Research*, **109**, B06406. DOI: 10.1029/2003JB002809.
- BIRD, P. 1991. Lateral extrusion of lower crust from under high topography, in the isostatic limit. *Journal of Geophysical Research*, **96**, 10275–10286.
- BUCK, W. R. & SOKOUTIS, D. 1994. Analogue model of gravitational collapse and surface extension during continental convergence. *Nature*, **369**, 737–740.
- CLARK, M. K. & ROYDEN, L. H. 2000. Topographic ooze: Building the eastern margin of Tibet by lower crustal flow. *Geology*, **28**, 703–706.
- GEMMER L., INGS, S., MEDVEDEV, S. & BEAUMONT, C. 2004. Salt tectonics driven by differential sediment loading: Stability analysis and finite element experiments. *Basin Research*, **16**, 199–218. DOI: 10.1111/j.1365-2117.2004.00229.x.
- GLEASON, G. C. & TULLIS, J. 1995. A flow law for dislocation creep of quartz aggregates determined with the molten salt cell. *Tectonophysics*, **247**, 1–23.
- JAMIESON, R. A., BEAUMONT, C., MEDVEDEV, S. & NGUYEN, M. H. 2004. Crustal channel flows: 2. Numerical models with implications for metamorphism in the Himalayan-Tibetan Orogen. *Journal of Geophysical Research*, **109**, B06407. DOI: 10.1029/2003JB002811.
- JOHANNES, W. & HOLTZ, F. 1996. *Petrogenesis and Experimental Petrology of Granitic Rocks*. Springer-Verlag, Berlin.
- KAUFMAN, P. S. & ROYDEN, L. H. 1994. Lower crustal flow in an extensional setting: Constraints from the Halloran Hills region, eastern Mojave Desert, California. *Journal of Geophysical Research*, **99**, 15723–15739.
- LEHNER, F. K. 2000. Approximate theory of substratum creep and associated overburden deformation in salt basins and deltas. In: LEHNER, F. K. & URAI, J. L. (eds) *Aspects of Tectonic Faulting*. Springer-Verlag, Berlin, 21–47.
- MEDVEDEV, S. 2002. Mechanics of viscous wedges: Modeling by analytical and numerical approaches. *Journal of Geophysical Research*, **107**, B6, 10.1029/2001JB000145.
- MEDVEDEV, S. E. & PODLADCHIKOV, YU. YU. 1999. New extended thin sheet approximation for geodynamic applications – II. 2D examples. *Geophysical Journal International*, **136**, 586–608.
- ÖZİŞİK, M. N. 1980. *Heat Conduction*. John Wiley, New York.
- ROYDEN, L. H. 1996. Coupling and decoupling of crust and mantle in convergent orogens: Implications for strain partitioning in the crust. *Journal of Geophysical Research*, **101**, 17679–17705.
- SHEN, F., ROYDEN, L. H. & BURCHFIEL, B. C. 2001. Large-scale crustal deformation of the Tibetan Plateau. *Journal of Geophysical Research*, **106**, 6793–6816.
- VANDERHAEGHE, O., MEDVEDEV, S., BEAUMONT, C., FULLSACK, P. & JAMIESON, R. A. 2003. Evolution of orogenic wedges and continental plateaux: insights from crustal thermal-mechanical models overlying subducting mantle lithosphere. *Geophysical Journal International*, **153**, 27–51.