

Polarization analysis in the wavelet domain based on the adaptive covariance method

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Accepted 2007 February 21. Received 2007 January 29; in original form 2005 November 14

SUMMARY

We present an improved method for computing polarization attributes of particle motion from multicomponent seismic recordings in the time–frequency domain by using the continuous wavelet transform. This method is based on the analysis of the covariance matrix. We use an approximate analytical formula to compute the elements of the covariance matrix for a time window which is derived from an averaged instantaneous frequency of the multicomponent record. The length of the time-window is automatically and adaptively set to match the dominant period of the analysing wavelet at each time–frequency point. Then the eigenparameters are estimated for each time–frequency point without interpolation. With these key features, our method provides a suitable approach for polarization analysis of dispersive signals or overlapping seismic arrivals in multicomponent seismic data. For polarization analysis in the time domain, we show that the proposed method is consistent with existing polarization analysis methods. We apply the method to real data sets from exploration and earthquake seismology to illustrate some filtering applications and wave type characterizations.

Key words: covariance method, multicomponent signal, polarization attributes, wavelet transform.

1 INTRODUCTION

The analysis of multicomponent recordings can provide an estimate of polarization properties of seismic arrivals. Knowledge of the polarization state can be used to distinguish between surface wave arrivals and body waves, or more generally for the purpose of wavefield decomposition (Flinn 1965; Kanasewich 1981; Jurkevics 1988; Jackson *et al.* 1991). Other applications of interest include anisotropy study through the investigation of shear wave splitting and the design of filters to improve the signal-to-noise ratios in seismic sections.

Frequency-dependent measurements, or time–frequency analysis (TFR), are particularly suitable for resolving seismic events with different spectral contents that overlap in time and for the analysis of transient signals. This analysis consists of examining the variation of the frequency content of a signal with time. To take advantage of these properties, time–frequency representations can be incorporated in polarization analysis (Soma *et al.* 2002; Diallo *et al.* 2005, 2006b; Pinnegar 2006).

Several methods are proposed in the literature for polarization analysis of multicomponent seismic data. Some of these methods allow the determination of the polarization attributes directly from the multicomponent signal (Rene *et al.* 1986; Morozov & Smithson 1996; Schimmel & Gallart 2003; Zhang *et al.* 2003; Diallo *et al.* 2006b), while others are based on the analysis of the covariance matrix of multicomponent recordings and principal component analysis using singular value decomposition (Kanasewich 1981; Vidale 1986; Park *et al.* 1987; Jurkevics 1988; Jackson *et al.* 1991; de Franco & Musacchio 2001).

For these methods, one needs to select a time-window where the analysis is performed. This step is critical to the success of the polarization attributes derivation. For multicomponent data with dispersive surface waves and overlapping seismic arrivals, the selection of the window length becomes more complicated. In such a case, obtaining interpretable attribute estimates from polarization analysis requires the separation of overlapping events. Then, for each resolved event, the window length should be adaptively selected to match the period of the dominant signal. Similar approaches for adaptively choosing appropriate window lengths have been used in the context of detection algorithms based on STA/LTA criteria (Withers *et al.* 1998; Bai & Kennett 2001).

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Diallo *et al.* (2006a) have proposed a method for polarization analysis in the time domain where the constraint of selecting the time window for the analysis has been relaxed by using an adaptive approach for its determination. In this paper, we extend this method to the time–frequency domain to derive polarization attributes with explicit dependence on frequency. We provide approximate analytical formulae for computing the entries of the covariance matrix in which the time window is automatically and adaptively determined.

The novelty in the present polarization method resides in the combined use of time–frequency representation along with an adaptive method of estimating the window for the covariance matrix. In this method, approximate analytical expressions for the elements of the covariance matrix are given which simplify their computation. Thus, besides the explicit dependence of attributes on frequency, the method provides a computationally efficient way to estimate the covariance matrix elements and determine the eigenparameters at each time and frequency.

We explicitly relate the polarization attributes to the wavelet coefficients. This enables us to design wavefield separation algorithms that operate entirely in the time–frequency domain. This is particularly important for the separation of body waves from surface waves, which are dispersive and space-variant in heterogeneous media. In the methodology developed hereafter, other TFR such the Gabor transform or S-Transform (Stockwell *et al.* 1996) could be used as a substitute to the CWT. The only difference in the polarization results will be those inherent to the difference in resolution between the different TFR. For instance, it is well known that because of its scalable analysis window, CWT provides a better frequency resolution than the Gabor transform, which uses a window with fixed width.

This paper is organized in the following manner. In the next section, we give an overview of the continuous wavelet transform. Then, we briefly introduce the standard covariance analysis method. Afterwards we present the extension of the adaptive covariance method to the CWT domain and elaborate on the consistency of this method with other techniques for polarization analysis. Next, we show that our present method provides a general framework for polarization analysis from which other methods (Rene *et al.* 1986; Morozov & Smithson 1996) can be derived. Finally using experimental data, we show how our method can be used for surface wave filtering which is an important issue in exploration seismology. We subsequently present a second example from earthquake seismology where we focus on the derivation of the 3-D instantaneous polarization attributes and on the use of these parameters in various filtering procedures.

2 INTRODUCTION TO THE CONTINUOUS WAVELET TRANSFORM

There exist at least two different perspectives to wavelet analysis. The first one is the interpretation of the wavelet transform as a time–frequency analysis tool. The second approach uses the wavelet analysis as a mathematical microscope, and is therefore, closely linked to approximation theory (Holschneider 1995). Our present work focuses on the first approach to wavelet analysis.

The wavelet transform of a signal $S(t) \in L^2(\mathbb{R})$ with respect to a mother wavelet $g(t)$ is the set of L^2 –scalar products of all dilated and translated wavelets with an arbitrary signal to be analysed:

$$\mathcal{W}_g S(t, a) = \langle g_{t,a}, S \rangle = \int_{-\infty}^{+\infty} \frac{1}{a} g^* \left(\frac{\tau - t}{a} \right) S(\tau) d\tau, \quad a \in \mathbb{R}, t \in \mathbb{R},$$

where $g_{t,a}(\tau) = \frac{1}{a} g[(\tau - t)/a]$ is generated from $g(\tau)$ through dilation a and translation t . The symbol $(.)^*$ denotes the complex conjugate. The wavelet $g(t)$ is assumed to be a function which is well localized in the time and frequency domains and obeys the oscillation condition $\int_{-\infty}^{+\infty} g(t) dt = 0$.

For the analysis of seismic signals, it is convenient to use the complex Morlet wavelet which can be written as (Holschneider 1995)

$$g(t) = e^{2\pi i f_0 t} e^{-\frac{t^2}{2\sigma^2}},$$

with its Fourier transform

$$\hat{g}(\omega) = \sigma e^{-\frac{\sigma^2(\omega - 2\pi f_0)^2}{2}},$$

where σ describe the variance of the wavelet.

The signal $S(t)$ can be recovered from its wavelet transform as

$$S(t) = \mathcal{M}_m \mathcal{W}_g S(t, a) = \frac{1}{C_{g,m}} \int_{-\infty}^{\infty} \int_{-\infty}^{+\infty} \frac{1}{a^2} m \left(\frac{t - \tau}{a} \right) \mathcal{W}_g S(\tau, a) d\tau da,$$

$$C_{g,m} = \int_0^{+\infty} [\hat{g}^*(\omega) \hat{m}(\omega) + \hat{g}^*(-\omega) \hat{m}(-\omega)] \frac{d\omega}{\omega},$$

where $m(t)$ is the wavelet, used for the inverse wavelet transform $\mathcal{M}_m$, and ω is the angular frequency. In the general case, the wavelet $m(t)$ can be different from the direct transform wavelet $g(t)$.

The wavelet transform can be expressed in terms of the Fourier transform $\hat{S}(f)$ of $S(t)$ as

$$\mathcal{W}_g S(t, a) = \int_{-\infty}^{+\infty} \hat{g}^*(a\zeta) e^{2\pi i t \zeta} \hat{S}(\zeta) d\zeta,$$

and the inverse of the scale $1/a$ may be associated with a frequency measured in units of the central frequency of $g(t)$. If the central frequency of the wavelet is assumed to be f_0 , each scale a can be related to the physical frequency f by $a = f_0/f$. Therefore, if we select a wavelet with a unit central frequency, it is possible to obtain the physical frequency directly by taking the inverse of the scale.

For the inverse wavelet transform, we might choose the δ -function for the wavelet $m(t)$, which then gives us a rather simple reconstruction formula

$$S(t) = \frac{1}{C_{g,\delta}} \int_{-\infty}^{+\infty} \mathcal{W}_g S(t, a) \frac{da}{a}.$$

3 STANDARD COVARIANCE METHOD

Let us consider the vector valued signal $\mathbf{S}(t) = [S_x(t), S_y(t), S_z(t)]^T$. We assume that this signal represents a three-component seismic recording. Here we associate $S_x(t)$, $S_y(t)$, $S_z(t)$ with the usual radial (inline), transverse, and vertical components of a triaxial recording. Polarization analysis of $\mathbf{S}(t)$ can be carried out by eigenanalysis of the cross-energy matrix $\mathbf{M}$, defined from the data vector as (Flinn 1965; Kanasevich 1981; Jurkevics 1988; Jackson *et al.* 1991)

$$\mathbf{M}(\xi) = \begin{bmatrix} I_{xx}(\xi) & I_{xy}(\xi) & I_{xz}(\xi) \\ I_{xy}(\xi) & I_{yy}(\xi) & I_{yz}(\xi) \\ I_{xz}(\xi) & I_{yz}(\xi) & I_{zz}(\xi) \end{bmatrix}.$$

In the equation above

$$I_{km}(\xi) = \frac{1}{T} \int_{\xi-T/2}^{\xi+T/2} [S_k(\tau) - \mu_k(\xi)][S_m(\tau) - \mu_m(\xi)] d\tau, \quad k, m = (x, y, z).$$

The above covariance matrix is computed within a time window of length T and around its time centre ξ ; $\mu_k(\xi)$ is the mean value of the signal component S_k in the window T .

The eigenanalysis performed on $\mathbf{M}(\xi)$ yields the principal component decomposition of the energy for the time window T . Such a decomposition produces three eigenvalues $\lambda_1(\xi) \geq \lambda_2(\xi) \geq \lambda_3(\xi)$ and three corresponding eigenvectors $\mathbf{v}_k(\xi)$ that fully characterize the magnitudes and directions of the principal components of the ellipsoid that approximates the particle motion in the considered time window:

- (i) the major half-axis $\mathbf{R}(\xi) = \sqrt{\lambda_1(\xi)} \frac{\mathbf{v}_1(\xi)}{\|\mathbf{v}_1(\xi)\|}$,
- (ii) the minor half-axis $\mathbf{r}(\xi) = \sqrt{\lambda_3(\xi)} \frac{\mathbf{v}_3(\xi)}{\|\mathbf{v}_3(\xi)\|}$,
- (iii) the second minor half-axis $\mathbf{r}_s(\xi) = \sqrt{\lambda_2(\xi)} \frac{\mathbf{v}_2(\xi)}{\|\mathbf{v}_2(\xi)\|}$,
- (iv) the reciprocal ellipticity $\rho(\xi) = \|\mathbf{r}_s(\xi)\|/\|\mathbf{R}(\xi)\|$,
- (v) the minor reciprocal ellipticity $\rho_1(\xi) = \|\mathbf{r}(\xi)\|/\|\mathbf{r}_s(\xi)\|$,
- (vi) the signed reciprocal ellipticity $\rho_s(\xi) = \text{sign}([\mathbf{v}_1 \otimes \mathbf{v}_2] \cdot \mathbf{v}_3) \rho(\xi)$,
- (vii) the dip angle $\delta(\xi) = \arctan(\sqrt{v_{1,x}(\xi)^2 + v_{1,y}(\xi)^2}/v_{1,z}(\xi))$,
- (viii) the azimuth $\alpha(\xi) = \arctan(v_{1,y}(\xi)/v_{1,x}(\xi))$,

where bold character indicates vectorial quantity, $\otimes$ and $\cdot$ denote the vectorial cross and inner product, respectively. To effectively use the standard covariance method for wavefield separation, one needs first to select the length of the time window T . Selecting this window may not be easy, especially when part of the signal under investigation is of a dispersive nature. We overcome this limitation by using the adaptive covariance approach that we are extending to the frequency domain in the next section.

4 ADAPTIVE COVARIANCE METHOD IN THE TIME-FREQUENCY DOMAIN

Let $\mathcal{W}_g \mathbf{S}(t, a)$ be the componentwise calculated progressive wavelet spectra ($a > 0$, wavelet is progressive) of the multicomponent signal $\mathbf{S}(t)$.

We denote $\Omega_k(t, a)$ as the instantaneous frequency function for each component k at each scale a at time t . This function is defined as follows:

$$\Omega_k(t, a) = \frac{\partial}{\partial t} \arg \mathcal{W}_g S_k(t, a).$$

To avoid the numerical derivatives in the above expression, it is possible to derive an exact analytical formula for the computation of $\Omega_k(t, a)$ that involves the wavelet transforms of the analysed signal with respect to the wavelet $g(t)$ and its derivatives $g'(t)$. This approach is widely used for instantaneous frequency estimate in Gabor transform analysis (Chassande-Mottin 1998; Borum & Jensen 1999). We adapt the methodology to the case of wavelet transform. The instantaneous frequency is then computed as follows:

$$\Omega_k(t, a) = -\Im \left[\frac{\mathcal{W}_{g'} S_k(t, a)}{\mathcal{W}_g S_k(t, a)} \right],$$

where $\Im(\cdot)$ denotes the imaginary part of the complex argument. For values of $\Omega_k(t, a)$ to be meaningful, the computation with the expressions above should be restricted to points in the (t, a) plane where the energy of the signal exceed some threshold ε_n above the background noise level: $\|\mathcal{W}_g S_k(t, a)\| > \varepsilon_n$.

A local approximation of each voice $S_k(t + \tau, a)$ of the wavelet transform $\mathcal{W}_g S_k(t, a)$ with scale a around time t can be written with the help of the modulus and phase of the wavelet coefficients as

$$S_k(t + \tau, a) \simeq |\mathcal{W}_g S_k(t, a)| \cos [\Omega_k(t, a)\tau + \arg \mathcal{W}_g S_k(t, a)]. \quad (1)$$

Using this approximation for each component $\mathcal{W}_g S_k(t, a)$, the cross-energy matrix $\mathbf{M}$ in the time–frequency domain can be constructed as

$$\mathbf{M}(t, a) = \begin{bmatrix} I_{xx}(t, a) & I_{xy}(t, a) & I_{xz}(t, a) \\ I_{xy}(t, a) & I_{yy}(t, a) & I_{yz}(t, a) \\ I_{xz}(t, a) & I_{yz}(t, a) & I_{zz}(t, a) \end{bmatrix},$$

$$I_{km}(t, a) = \frac{1}{T_{km}(t, a)} \int_{-T_{km}(t, a)/2}^{T_{km}(t, a)/2} [S_k(t + \tau, a) - \mu_{km}][S_m(t + \tau, a) - \mu_{mk}] d\tau = |\mathcal{W}_g S_k(t, a)| |\mathcal{W}_g S_m(t, a)| \cdot$$

$$\times \left\{ \text{sinc} \left[\frac{\Omega_k(t, a) - \Omega_m(t, a)}{2} T_{km}(t, a) \right] \cos [\arg \mathcal{W}_g S_k(t, a) - \arg \mathcal{W}_g S_m(t, a)] \right.$$

$$\left. + \text{sinc} \left[\frac{\Omega_k(t, a) + \Omega_m(t, a)}{2} T_{km}(t, a) \right] \cos [\arg \mathcal{W}_g S_k(t, a) + \arg \mathcal{W}_g S_m(t, a)] \right\} - \mu_{km} \mu_{mk}, \quad (2)$$

where the mean values μ_{km} are defined as

$$\mu_{km} = \frac{1}{T_{km}(t, a)} \int_{-T_{km}(t, a)/2}^{T_{km}(t, a)/2} S_k(t + \tau, a) d\tau = \Re[\mathcal{W}_g S_k(t, a)] \text{sinc}[T_{km}(t, a) \Omega_k(t, a)/2]. \quad (3)$$

In the equation above, $\Re(\cdot)$ implies the real part of a complex variable, and $\text{sinc}(x)$ indicates the sine cardinal function. The matrix $\mathbf{M}(t, a)$ is defined for each time t and scale a ; thus its eigenanalysis yields the instantaneous polarization attributes in the time–frequency domain.

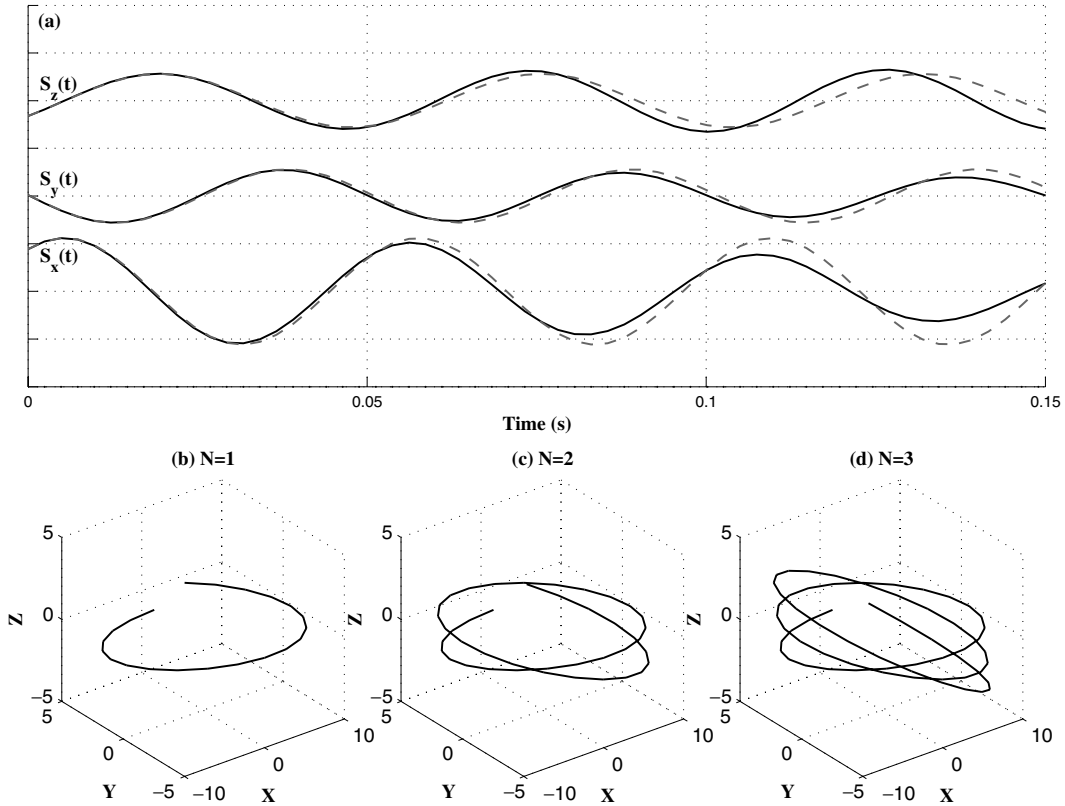


Figure 1. Illustration of the influence of increasing N in eqs (6) and (7) on the accuracy of the approximation for the signals. (a) Three synthetic signals $S_x(t)$, $S_y(t)$ and $S_z(t)$ (solid lines) and the corresponding approximation (dashed lines). (b–d) Hodograph plots for each segment of the approximated signals for the different values of N . For $N = 1$ the approximation is up to 0.05 s, for $N = 2$ the approximation extends to 0.1 s and for $N = 3$, the approximation covers the whole time axis. Note how the curves drift apart as N increases.

$T_{km}(t, a)$ represents the adaptive time window used in eqs (2) and (3) which is determined using the instantaneous frequencies at time t and scale a .

From eq. (1), we note that each voice in the wavelet transform is locally approximated by a cosine function with a period of $2\pi/\Omega_k(t, a)$. On one hand, this equation allows us to obtain an analytical formula to compute the elements of the covariance matrix at each time–frequency location. On the other hand, the different signal components often have different instantaneous frequencies in the case of real signals. Therefore, we need an approach to obtain an average time window $T_{km}(t, a)$ for two voices $\mathcal{W}_g S_k(t, a)$ and $\mathcal{W}_g S_m(t, a)$ having different frequencies $\Omega_k(t, a)$ and $\Omega_m(t, a)$.

Unfortunately, there is no mathematically exact a priori definition for the instantaneous polarization attributes of a multicomponent signal. Therefore, any attempt to produce one is usually arbitrary. From this observation, it follows that we cannot construct any rigorous criteria to compare the accuracy of different approaches for the time window estimation.

The way we choose our time window is motivated by the fact that for pure elliptically polarized signals, where $\Omega_x(t, a) = \Omega_y(t, a) = \Omega_z(t, a) = \Omega(t, a)$, we obtain the exact values of the instantaneous parameters using our approach. For example, we can choose $T_{km}(t, a)$ to be the same for all entries of the cross-matrix energy $\mathbf{M}(t, a)$ as

$$T_{km}(t, a) = \frac{6\pi N}{\Omega_x(t, a) + \Omega_y(t, a) + \Omega_z(t, a)} = \frac{2\pi N}{\Omega_{av}^{xyz}(t, a)}, \quad (4)$$

or define it in a specific manner for each entry as

$$T_{km}(t, a) = \frac{4\pi N}{\Omega_k(t, a) + \Omega_m(t, a)} = \frac{2\pi N}{\Omega_{av}^{km}(t, a)}. \quad (5)$$

In such a case, $T_{km}(t, a)$ from both eqs (4) and (5) reduces to $2\pi N/\Omega(t, a)$, which is an integral number times the signal period. Therefore, taking N as an integer ensures that the integration in eqs (2) and (3) is always performed over a window covering complete cycles of the signals that provide the polarization parameters for pure elliptically polarized signals precisely.

However, multicomponent signals where the phase of individual components are equal at all times is unrealistic, and for such a case the polarization analysis is trivial. In general, the instantaneous frequencies from the components are different, and $T_{km}(t, a)$, as defined in (4) and (5), with integer N will still represent an average period. At the same time, it is in principle possible to use real values for N in these equations. However, in this case, the obtained polarization parameters do not correspond to the polarization of pure elliptically polarized signals that we note by numerical simulations as an increase of local oscillations of polarization attributes.

The effect of integer-valued varying of the parameter N is shown in Fig. 1. Fig. 1(a) shows three signals $S_x(t)$, $S_y(t)$, $S_z(t)$ (solid line) and their respective local approximations using eq. (1) (dashed line). For this example we used slightly different instantaneous frequencies for $S_x(t)$, $S_y(t)$ and $S_z(t)$. The adaptive time window for these signals obtained from eq. (4) with $N = 1$ is approximately 0.05 s. In Fig. 1(a) we observe that up to 0.05 s on the time axis, the approximations fit the actual signals very well. From the signal curves and the hodograph plot in Fig. 1(b) we also see that all signals in this time interval have approximately undergone one full cycle. Next, we note that as N increases, the accuracy of the approximation to the signals decreases, but a structure that is more complex than an ellipse starts to build up from the hodograph in Fig. 1(b). For signals with polarization structure contained in a plane, it is enough to use $N = 1$ or 2. For signals with a more complex polarization structure, we can improve our ability to characterize an eventual third polarization direction with larger N , but it comes always at the expense of accuracy in approximating the signals.

4.1 Linking the adaptive covariance method to other existing methods

In this section, we show that the polarization methods by Rene *et al.* (1986) and Morozov & Smithson (1996) can be derived from the adaptive covariance method. Since those methods were initially developed in the time domain, we will here consider the $\mathbf{M}(t)$ in the time domain as given in Diallo *et al.* (2006a). This is achieved by dropping the variable a from (2) and replacing $\mathcal{W}_g S_k(t, a)$ by $C_k(t)$ which is the analytical signal associated to $S_k(t)$:

$$C_k(t) = S_k(t) + i\mathcal{H}S_k(t),$$

where $\mathcal{H}S(t)$ denotes the Hilbert transform.

First we consider the analysis of a two-component signal given by $\mathbf{S}(t) = [S_x(t), S_y(t)]^T$. Let $S_x(t)$ represent the horizontal component and $S_y(t)$ the vertical component. We also assume that the instantaneous frequencies of these components are equal: $\Omega_x(t) = \Omega_y(t) = \Omega(t)$. Then proceeding as previously, one obtains the following cross-energy matrix

$$\mathbf{M}(t) = \begin{bmatrix} |C_x(t)|^2 & |C_x(t)||C_y(t)| \cos[\arg C_x(t) - \arg C_y(t)] \\ |C_x(t)||C_y(t)| \cos[\arg C_x(t) - \arg C_y(t)] & |C_y(t)|^2 \end{bmatrix}.$$

The eigenanalysis of this matrix yields the following eigenvalues:

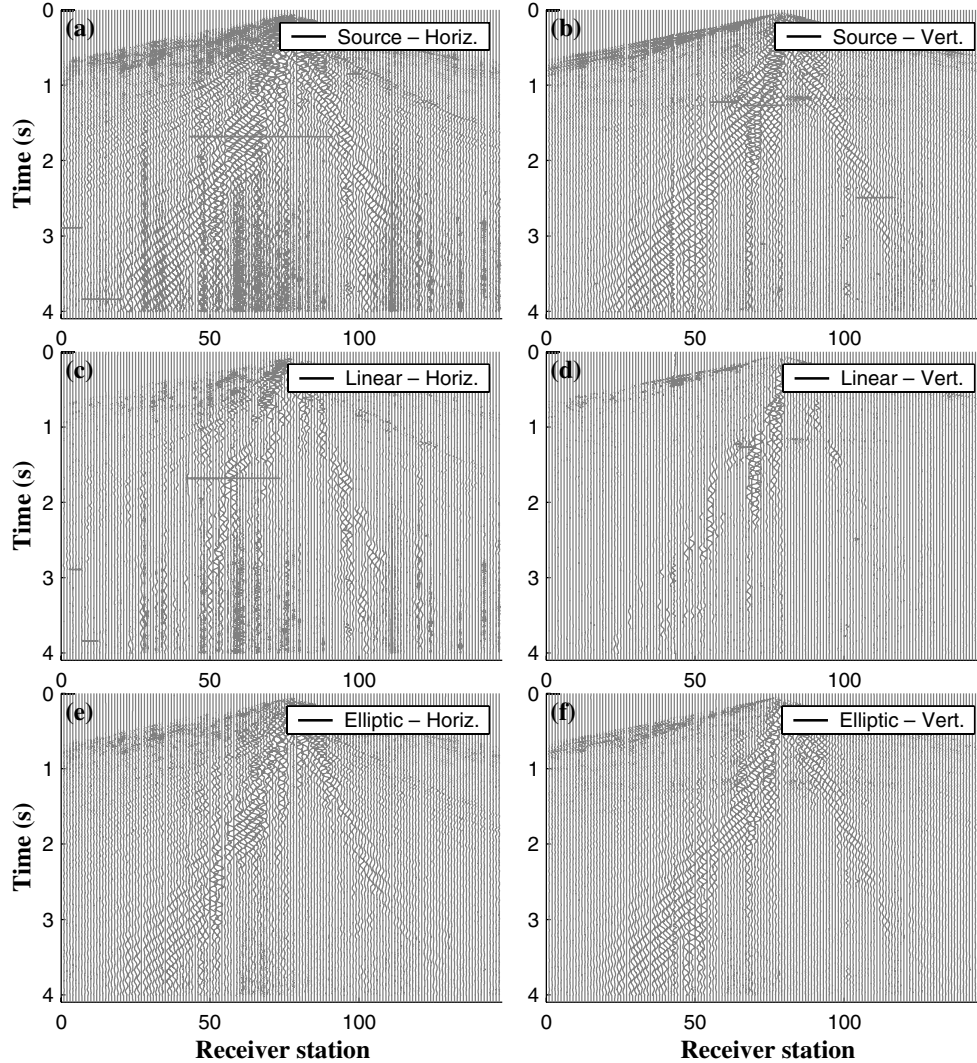
$$\lambda_{1,2}(t) = \frac{1}{2} [|C_x(t)|^2 + |C_y(t)|^2 \pm \sqrt{[|C_x(t)|^2 - |C_y(t)|^2]^2 + \{2|C_x(t)||C_y(t)| \cos[\Delta\phi(t)]\}^2}],$$

where $\Delta\phi(t) = \arg[C_x^*(t)C_y(t)]$.

These two eigenvalues correspond exactly to the semi-minor and semi-major axes of the polarization ellipse as given by eqs (8) and (9) in Rene *et al.* (1986).

Table 1. Classification of wave modes using the polarization attributes.

Polarization	Filter	P_ρ	P_θ
Linear and horizontal	F_{LH}	$[0, \rho_f] \cup [1 - \rho_f, 1]$	$[-\theta_f, \theta_f]$
Linear and vertical	F_{LV}	$[0, \rho_f] \cup [1 - \rho_f, 1]$	$[-\pi/2, -\theta_f] \cup [\theta_f, \pi/2]$
Elliptical and horizontal	F_{EH}	$[\rho_f, 1 - \rho_f]$	$[-\theta_f, \theta_f]$
Elliptical and vertical	F_{EV}	$[\rho_f, 1 - \rho_f]$	$[-\pi/2, -\theta_f] \cup [\theta_f, \pi/2]$

**Figure 2.** Real 2C shot gather from Fort Worth Basin: (a) horizontal component; (b) vertical component; (c and d) overall linearly polarized events and (e and f) overall elliptically polarized events.

Let us now consider the case of a three-component signal given by $\mathbf{S}(t) = [S_x(t), S_y(t), S_z(t)]^T$. Here again we assume the instantaneous frequencies of these components to be equal ($\Omega_x(t) = \Omega_y(t) = \Omega_z(t) = \Omega(t)$). In this case, the entries of the cross-matrix energy $\mathbf{M}(t)$ are given as

$$I_{km}(t) = |C_k(t)| |C_m(t)| \cos [\arg C_k(t) - \arg C_m(t)] = \Re [C_k(t) C_m^*(t)].$$

The eigenvalues of the matrix $\mathbf{M}(t)$ are related to the ellipticity parameters obtained from the adaptive covariance method as

$$\lambda_1(t) = \|\mathbf{R}\|, \quad \lambda_2(t) = \|\mathbf{r}\|, \quad \lambda_3(t) = 0.$$

Note that for any number of components (greater than three), if one assumes equal instantaneous frequency for all components, the eigenanalysis will give at most two eigenvalues that are different from zero. Therefore, with this assumption and the eigenanalysis of $\mathbf{M}(t)$, one can characterize, at most, an ellipse in 3-D space (it is not possible to assess the importance of the third eigenvalue which is needed

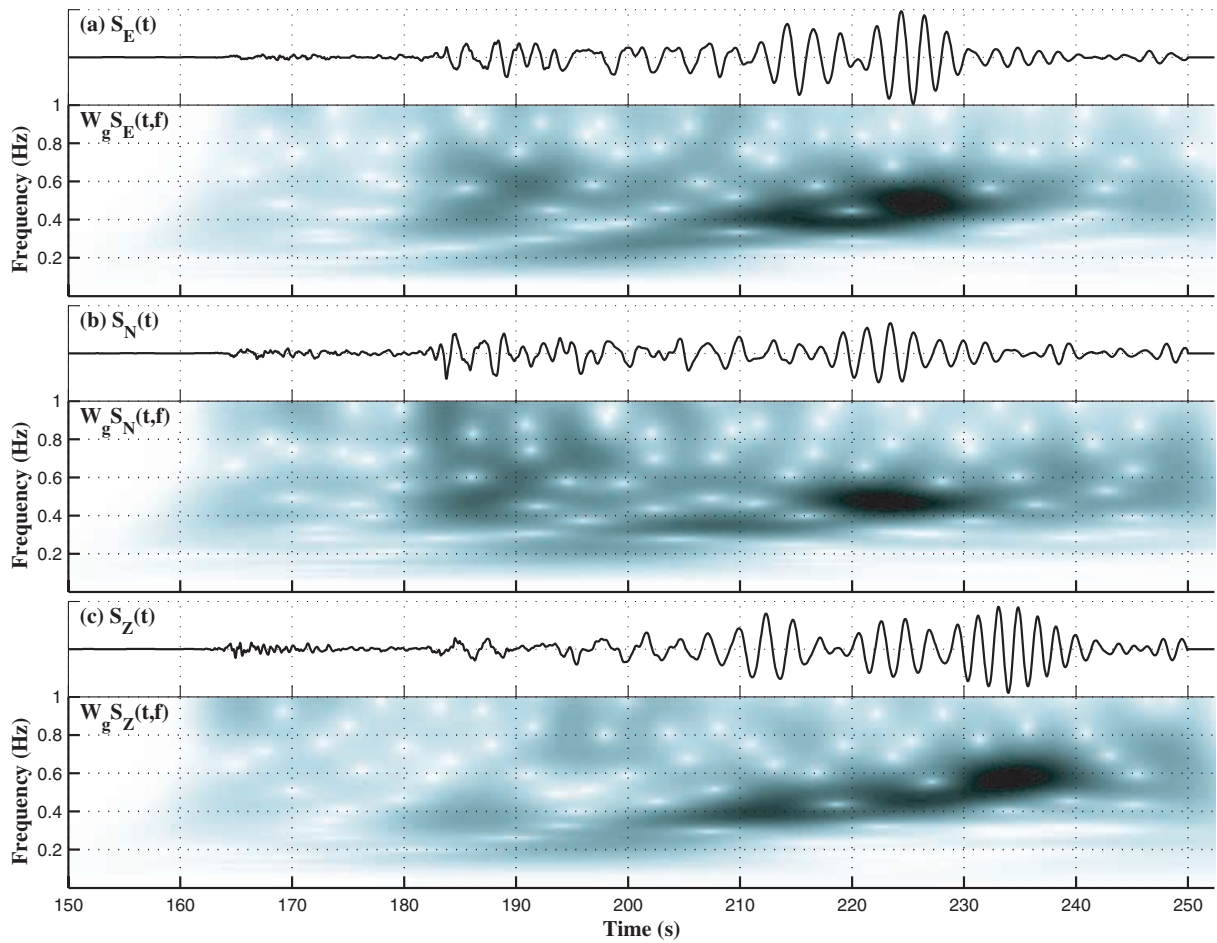


Figure 3. Three-component seismograms recorded at the GRA1 stations, generated by a salt mine collapse in Thuringen, Germany in 1989 and the continuous wavelet transforms of this seismograms performed componentwise: (a) east component, (b) north component and (c) vertical component.

for the characterization of the polarization ellipsoid). Polarization analysis with the present assumption yields the same results as those from Morozov's method (Morozov & Smithson 1996).

5 FILTERING ALGORITHM

With the extension of the adaptive covariance method to the wavelet domain, we can construct filtering algorithms to separate different wave types based on the instantaneous attributes. For instance, we could construct an algorithm to separate body waves from surface waves (Rayleigh wave) by a combination of constraints posed on the range of the reciprocal ellipticity $\rho(t, a)$ and the rise angle $\theta(t, a) = \pi/2 - \delta(t, a)$. Formally the algorithm can be represented as

$$S_k^f(t) = \Re[\mathcal{M}_m \mathcal{F}_e \mathcal{W}_g S_k(t, a)], \quad a > 0,$$

$$\mathcal{F}_e(t, a) = \begin{cases} \mathcal{W}_g S_k(t, a) & \text{for } \rho(t, a) \in P_\rho \text{ and } \theta(t, a) \in P_\theta, \\ 0 & \text{otherwise,} \end{cases}$$

where $\mathcal{F}_e$ is the filter operator of the wavelet-spectrum applied to each component of the source signal. The sets P_ρ and P_θ define the range of ρ and θ which are kept in the filtered signal. The range of P_ρ and P_θ can be selected in such a way to obtain the desired wave type.

In Table 1, we summarize filter settings that can be used to detect signals with specific polarization. We have already used a similar filtering approach for 2C seismic data (Diallo *et al.* 2006b). However, the present method has the advantage of being applicable to unlimited number of components.

6 DATA EXAMPLES—APPLICATION TO SIGNAL FILTERING

6.1 Real 2C seismic data

We test the performance of the filtering algorithm on real 2C seismic data from Fort Worth Basin USA (Gherasim *et al.* 2004) shown in

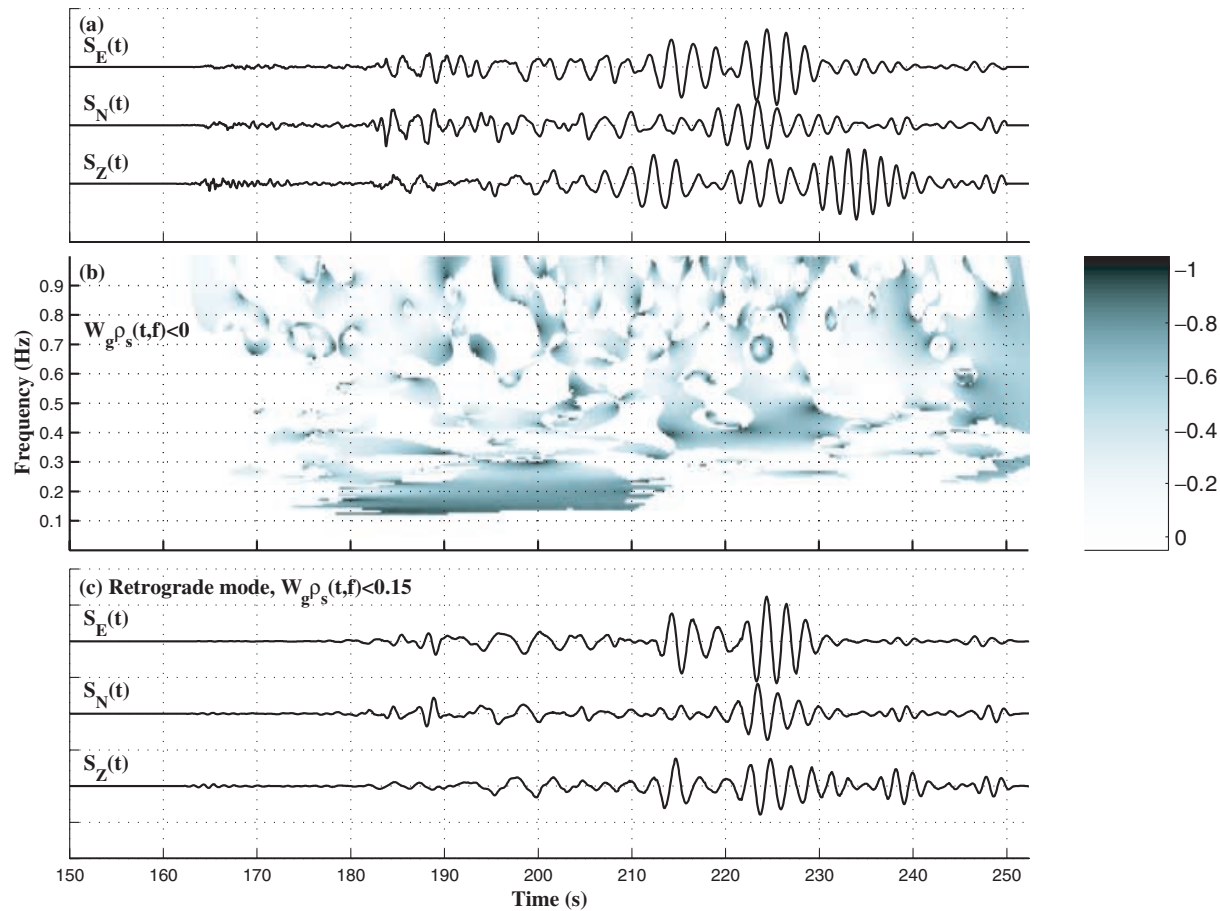


Figure 4. A filtration based on the signed ellipticity ratio: (a) input data, (b) negative part of the signed ellipticity ratio in time–frequency plot and (c) extracted retrograde multicomponent signal with the filtration condition $W_g \rho_s(t, f) < -0.15$.

Figs 2(a)–(b). These are raw data that were not subject to any prior pre-processing. We evaluate the filter to test its ability for separating the Rayleigh waves from other wave types. This is an important application in oil exploration where Rayleigh wave are usually considered as noise that interferes with and corrupts reflection events, the identification and interpretation of which is the primary purpose of seismic processing.

In our filtering approach, we deliberately decide to recover not only the ‘signal’ but the ‘noise’ as well. The reason is that for near surface characterization, Rayleigh waves are increasingly used as a cost effective method for a quick estimate of an average shear velocity profile for engineering purposes (Beatty & Schmitt 2003). With this idea in mind, we filtered the raw shot gathers twice with different settings in the combination and the range of the ellipticity ratio and the rise angle.

To extract the predominantly linearly polarized wave types (body waves), we set the filter parameters as $\rho(t, a) \in [0, 0.15] \cup [0.85, 1]$. The resulting filtered shot gathers are shown in Figs 2(c) and (d). To extract the wavefield that includes the Rayleigh wave arrivals, we tune the filter to ‘pass’ only signals with some ellipticity delimited by the pre-selected range for $\rho(t, a) \in [0.15, 0.85]$ without restriction on the range of $\theta(t, a)$. Since we did not delimit the range of $\theta(t, a)$, the extracted wavefield may contain various elliptically polarized wave types with major axes pointing in different directions. The filtered shot gathers for this filter setting are shown in Figs 2(e) and (f). From these figures, one can clearly identify the energetic low frequency arrivals which could be associated to the Rayleigh waves. Note that these arrivals are not present in Figs 2(c)–(d) where the filter was tuned to pass only linearly polarized events. With the flexibility in our filter setting, we can perform directional wave filtering or wavefields separation for any number of components.

6.2 Real 3C data from a regional earthquake

We illustrate the importance of the application of the method developed in this contribution to three-component broad-band data. Here we use a set of high quality seismograms recorded at the GRA1 station of the GRF array in northern Bavaria (Germany) shown in Fig. 3. The event [origin time: 13-mar-1989 13:02, lat: 50.7198N, lon: 9.9112E, $h = 0.7$ km, $M_L = 5.6$, On-line Bulletin (2001)] was triggered by the collapse of a salt mine in Thüringen in 1989 in a distance of 148 km and a backazimuth of 321.2° against North from station GRA1. Fig. 3 shows also the wavelet transforms of the different components, respectively.

The seismograms start with the P -wave onset (about 165 s relative time, time axis starts at 1989 March 13, 13:00), which at this distance is composed of the head wave P_n and the direct P wave P_g in regional velocity models used for localization (Schlittenhardt 1999). About

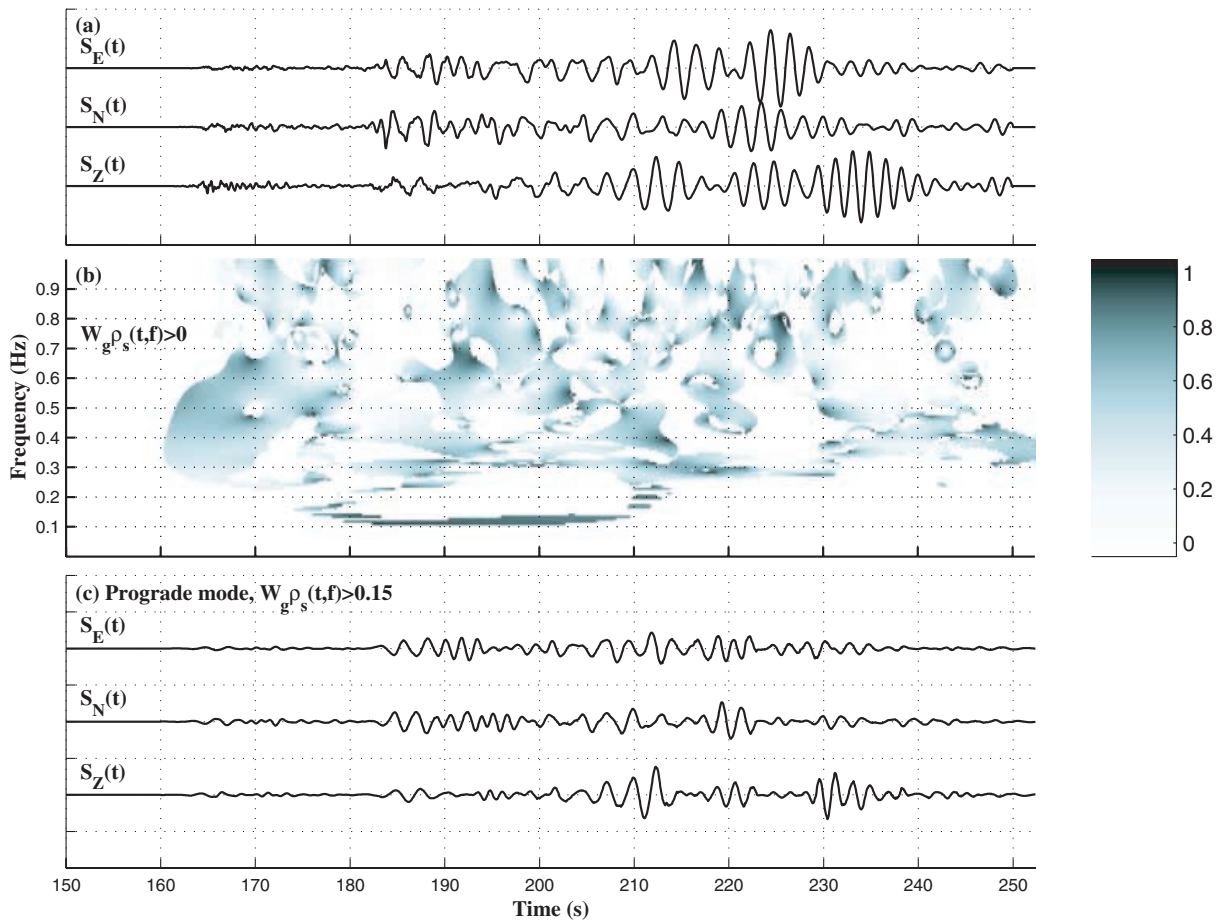


Figure 5. A filtration based on the signed ellipticity ratio: (a) input data, (b) positive part of the signed ellipticity ratio in time–frequency plot and (c) extracted prograde multicomponent signal with the filtration condition $W_g \rho_s(t, f) > 0.15$.

20 s later, shear waves emerge with strong amplitudes on the horizontal components. In the time window between 180 and 240 s, the arrival of the Rayleigh wave on the vertical component $S_z(t)$ is observed. Because the source is a collapse event, we can expect that *SH* waves are, in general, not strongly exaggerated. The amplitude of the wavelet coefficients in Fig. 3 shows that dominant frequencies are larger than 0.1 Hz contribute to the seismograms. Between 180 and 240 s, the amplitudes for the lowest frequencies show clear dispersion with low frequencies (0.15 Hz) arriving early, and higher frequencies (0.6 Hz) arriving late (see fig. 1 in Hadiouche *et al.* (1991), for a classical group velocity analysis of this event for station GRA1).

As an application of the method, Fig. 4(b) shows the signed ellipticity ratio corresponding to retrograde particle motion. The wavefield in the abovementioned time and frequency windows has a retrograde motion and can, therefore, be safely identified with the fundamental mode of the Rayleigh wave. It is now straightforward to use the inverse wavelet transform to suppress or enhance a specific polarization characteristic [in 4(b) all $W_g \rho_s(t, f) < -0.15$ are selected to enhance the specifically retrograde components]. In the resulting traces (Fig. 4c), lower frequency components of the Rayleigh wave fundamental mode shows up clearly between 190 and 205 s.

Fig. 5 shows the signed ellipticity ratio and polarization filtered time traces for prograde particle motions. The first overtone of the Rayleigh wavefield is expected to arrive directly after the *S* wave. The waves observed at about 210 s relative time (*Z* component) most likely correspond to the fundamental mode of the Rayleigh wave, but are in time and frequency nearer to wavefield components polarized off the great circle path which arrive slightly before (see Fig. 6). Note also the amplitudes following the *P* wave onset with quite high ellipticity value, either due to *P*–*SV* conversions at interfaces (Krüger 1994) and/or due to the presence of a leaky mode P_L . The dark line between at about 0.1 Hz and between 180 and 210 s in Fig. 5(b) is caused by the very low amplitudes of the wavelet coefficients (see Fig. 3) and an artefact.

Fig. 6 shows the deviations of the particle motions from the theoretically expected direction (great circle path). While most of the early parts (up to 200 s relative time) of the seismograms correspond to waves which have travelled along the great circle path with only moderate deviation, we can identify between 200 and 210 s a wavefield component with characteristics of a *SH*-type wave (Love wave). The filtered seismograms in Fig. 6(c) show how the waves approaching the station along the great circle path can be easily isolated from scattered phases in the coda.

Finally, Fig. 7 shows a filtering example where only dip angles are retained which correspond to horizontally polarized particle motion.

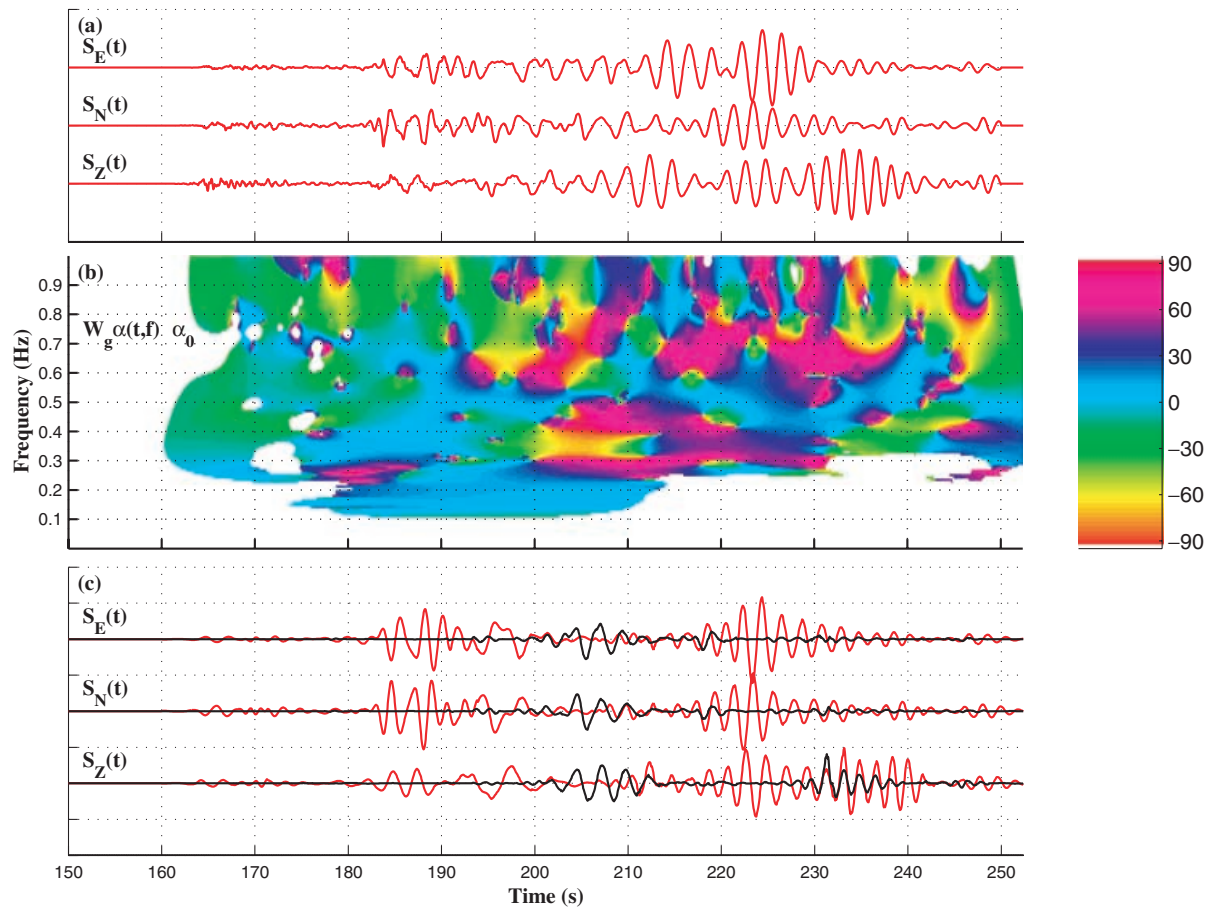


Figure 6. A filtration based on the azimuth: (a) input data, (b) deviation of the particle motions from the theoretically expected direction $\alpha_0 = 128.8^\circ$ and (c) extracted multicomponent signal with the filtration conditions $(W_g \alpha(t, f) - \alpha_0) \in [-20^\circ, 20^\circ] \cup [70^\circ, -70^\circ]$ for the red line and $(W_g \alpha(t, f) - \alpha_0) \in [25^\circ, 65^\circ] \cup [-65^\circ, -25^\circ]$ for the black line.

7 CONCLUSIONS AND DISCUSSION

In this paper, we consider the extension of the adaptive covariance method to the time–frequency domain. We discuss the application of the method to exploration seismology by designing a filter for wavefield decomposition that enables the separation of body waves from ground-roll. For application in earthquake seismology, we show how the extracted polarization attributes can be used as effective tool to characterize wave types present in actual multicomponent records.

We establish the consistency of our method with other techniques (Rene *et al.* 1986; Morozov & Smithson 1996). We found that when the instantaneous frequencies are the same for all components, our method produces the same results as those in (Morozov & Smithson 1996) in terms of polarization parameters. In that regards, our method is more general in scope as it allows the characterization of the polarization ellipsoid, whereas Morozov's method is by design restricted to the characterization of an ellipse in 3-D space.

The adaptive aspect of our method comes from the fact that we locally adjust the window size for the covariance matrix computation to the period of the signal. Obviously, our method can be applied for any number of components. This representation allows the detection of dispersion in the polarization attributes which can be further exploited to infer some physical characteristics of the medium under investigation. Moreover, this representation offers the ability to distinguish between attributes belonging to different coherent events that may overlap in time but with different frequency content separated by time-dependent frequency cut-offs.

CWT gives a suitable general framework for solving this type of problem; this approach is powerful and elegant, but not critical for understanding the results. Other TFR methods such as the Gabor transform or the S -transform can be used in our approach in general. The relative performance of polarization analysis from a different TFR approach is primarily controlled by the frequency resolution capability that motivated the use of CWT instead of the Gabor transform in the present work.

The novelty in the present polarization method resides in the combined use of time–frequency representation along with an adaptive method of estimating the window for the covariance matrix. Some advantages of our method are as follows.

(i) The derivation of analytical formulae to compute the elements of the covariance matrix at each time–frequency location. Thus, we do not have to deal with edge effects or interpolation to obtain the eigenparameters at each point as it may be the case with the standard covariance method.

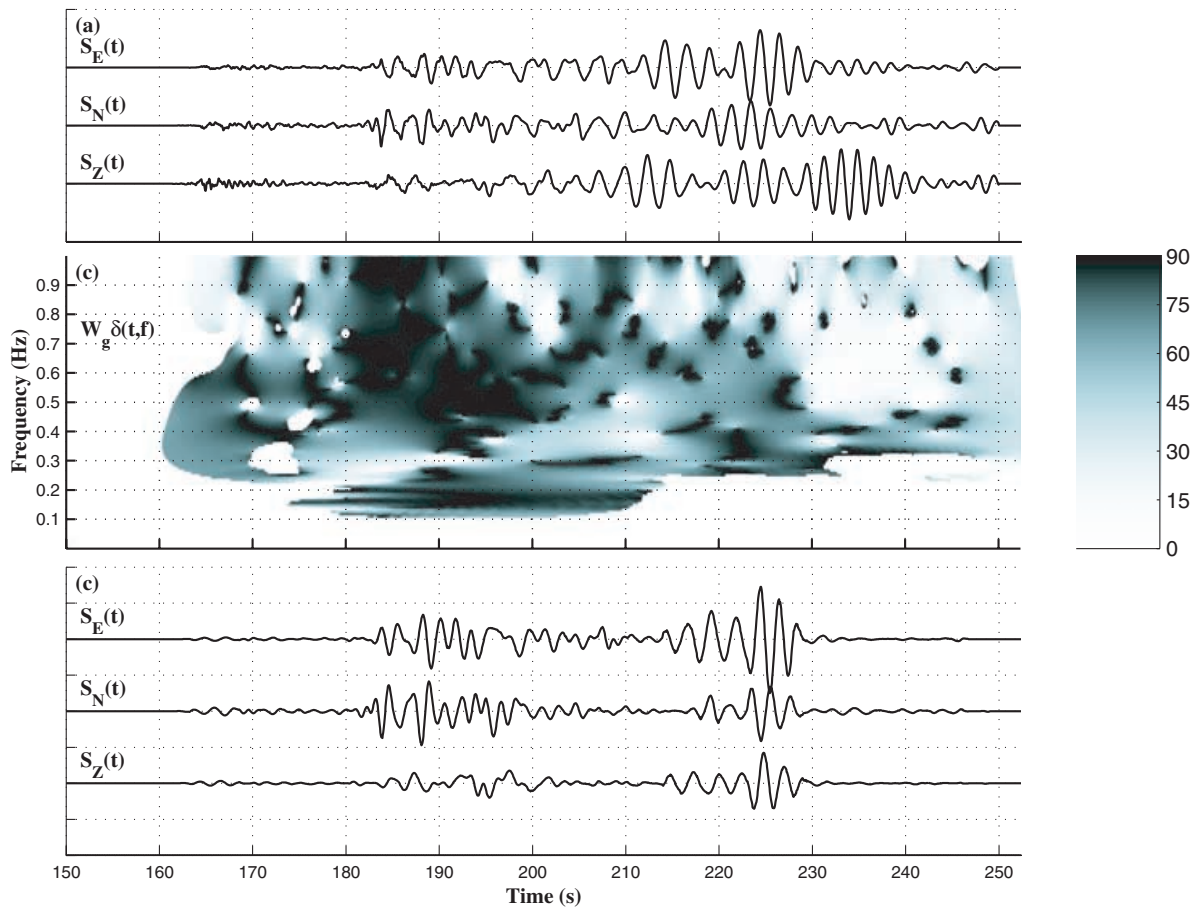


Figure 7. A filtration based on the dip attribute: (a) input data, (b) the dip attribute given in time-frequency plot and (c) polarization filtered results of the input data to extract part of the multicomponent signal arriving at an apparent dip angle range defined by the condition $W_g \delta(t, f) \in [70^\circ, 90^\circ]$.

(ii) As shown in the example from three-component broad-band data, it is possible to extract all the attributes of the polarization ellipsoid. Visualization of these attributes in the time–frequency domain can be very helpful in practice for the interpretation of wave types.

(iii) The possibility for identifying and separating different wave types, which is achieved by designing filters that operate in the time–frequency domain as explained in Section 5. Attributes such as azimuth, dip and signed ellipticity can be also used to improve the filtering algorithms.

ACKNOWLEDGMENTS

This project is supported by a grant from the Deutsche Forschungsgemeinschaft (DFG) within the framework of the priority program SPP 1114 “Mathematical methods for time series analysis and digital image processing”. Kurt J. Marfurt from the Allied Geophysics Laboratory (University of Houston) provided the real data set from Fort Worth Basin. The Seismological Central Observatory Graefenberg (SZGRF) contributed the data for the seismological example. We are grateful to the reviewers for the constructive criticism that greatly helped improve the content of this manuscript.

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