

FAST TRACK PAPER

Absolute S -velocity estimation from receiver functions

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SUMMARY

We present a novel method to recover absolute S velocities from receiver functions.

For a homogeneous half-space the S velocity can be calculated from the horizontal slowness and the angle of surface particle motion for an incident P wave. Generally, the calculated S velocity is an apparent half-space value which depends on model inhomogeneity and P -waveform. We therefore, suggest to calculate such apparent half-space S velocities from low-pass filtered (smoothed) receiver functions using a suite of filter-parameters, T . The use of receiver functions neutralize the influence of the P -waveform, and the successive low-pass filterings emphasize the variation of S velocity with depth.

We apply this $V_{S,app.}(T)$ technique to teleseismic data from three stations: FUR, BFO and SUM, situated on thick sediments, bedrock and the Greenland ice cap, respectively. The observed $V_{S,app.}(T)$ curves indicate the absolute S velocities from the near surface to the uppermost mantle beneath each station, clearly revealing the different geological environments. Application of linearized, iterative inversion quantify these observations into $V_S(z)$ models, practically independent of the S -velocity starting model. The obtained models show high consistency with independent geoscientific results. These cases provide also a general validation of the $V_{S,app.}(T)$ method.

We propose the computation of $V_{S,app.}(T)$ curves for individual three-component broad-band stations, both for direct indication of the S velocities and for inverse modelling.

Key words: absolute velocity, apparent incidence angle, free surface, receiver function, S velocity.

1 INTRODUCTION

In applied receiver function analysis it is often stated that receiver functions are not sensitive to the absolute levels of the S velocity (e.g. Ammon *et al.* 1990; Kind *et al.* 1995; Schlindwein 2006; Tomlinson *et al.* 2006). In the present paper, we show that this is not entirely correct. We present a simple transform which clearly emphasizes the absolute S -velocity information present in receiver functions. The effect of the free surface on an incoming teleseismic P wave plays a key role in this method.

A plane P wave incident on a free surface is reflected as a P wave and a converted SV wave. The particle motion (or polarization) observed by a three-component seismograph on this surface is the superposition of the incoming and the two outgoing waves. As a result the apparent incidence angle ($\bar{i}_P$) defined by the surface particle motion is different from the true P wave incidence angle (i_P). The relation between true and apparent incidence angles was early quantified in Wiechert (1907, eq. 128) (see also e.g. Nuttli & Whitmore 1961) from which follows

$$\frac{\sin(\frac{1}{2}\bar{i}_P)}{V_S} = \frac{\sin(i_P)}{V_P} = p, \quad (1)$$

where V_P and V_S are compressional and shear velocities of the half-space and p the horizontal slowness (ray parameter) of the P wave. Eq. (1) can be rearranged to

$$V_S = \frac{\sin(\frac{1}{2}\bar{i}_P)}{p}, \quad (2)$$

which defines the half-space S velocity as a function of the observed apparent incidence angle and slowness of a P wave. No assumptions are made concerning V_P or V_P/V_S . Bostock & Rondenay (1999) derived an equivalent but less simple expression (their eq. A6) from the free-surface transfer matrix of Kennett (1991).

For general velocity distributions, the S velocity estimate using eq. (2) is an apparent half-space S velocity, $V_{S,app.}$, which depends on the S - and P -velocity structure of the subsurface, the incoming P -waveform and the event slowness and backazimuth.

We propose to neutralize the influence of the P -waveform on $V_{S,app.}$ by estimating $\bar{i}_P$ from (Z, R) receiver functions instead of raw (Z, R) data. Further we suggest to estimate $\bar{i}_P$ as a function of a low-pass (smoothing) filter-parameter, T , resulting in $V_{S,app.}(T)$ curves that emphasize the absolute S velocity variation with depth.

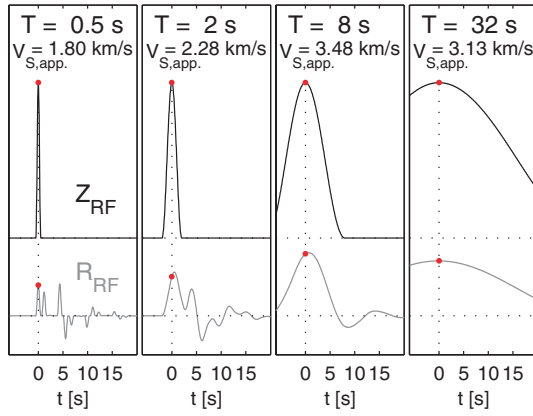


Figure 1. The synthetic (Z , R) receiver functions for the layer-over-half-space model in Fig. 2(a), low-pass filtered (smoothed) using four different filter-parameters, T . The values of T and the resulting $V_{S,app.}(T)$ estimate are written on each plot and the amplitudes of R_{RF} and Z_{RF} at $t = 0$ are marked with red dots.

2 $V_{S,app.}(T)$ ESTIMATION TECHNIQUE

The use of (Z , R) receiver functions in the estimation of $\bar{i}_P$ yields some advantages relative to estimation of $\bar{i}_P$ from raw records of (Z , R) particle motion. Receiver function estimation is the deconvolution of the Z component from the R component (R_{RF}) and Z deconvolved from itself (Z_{RF}) performed in either frequency domain (e.g. Ammon 1991) or time domain (e.g. Petersen *et al.* 1993). Deconvolution neutralizes the incoming P waves, which dominate Z , so that Z_{RF} is an approximate zero-phase spike with arrival instant at exactly $t = 0$. Therefore, $\bar{i}_P$ can be estimated as

$$\bar{i}_P = \arctan \left[\frac{R_{RF}(t=0)}{Z_{RF}(t=0)} \right]. \quad (3)$$

To estimate $\bar{i}_P$ as a function of the low-pass filter-parameter T , the (Z , R) receiver functions are smoothed in time domain with a squared-cosine shape of width T according to

$$h(t) = \begin{cases} \cos^2 \left(\frac{\pi t}{2T} \right) & \text{for } |t| < T \\ 0 & \text{for } |t| \geq T. \end{cases} \quad (4)$$

Only the amplitudes of the filtered (Z_{RF} , R_{RF}) at $t = 0$ are relevant (eq. 3). Therefore, $\bar{i}_P(T)$ can be computed efficiently as

$$\bar{i}_P(T) \equiv \arctan \left[\frac{\int_{-T}^T R_{RF}(\tau) \cos^2 \left(\frac{\pi \tau}{2T} \right) d\tau}{\int_{-T}^T Z_{RF}(\tau) \cos^2 \left(\frac{\pi \tau}{2T} \right) d\tau} \right]. \quad (5)$$

Note that, larger T implies more smoothing so that more and more multiples influence the values of the filtered receiver functions at $t = 0$ (see Fig. 1).

Combination of eqs (5) and (2) yields the $V_{S,app.}(T)$ estimate

$$V_{S,app.}(T) \equiv \frac{\sin \left[\frac{1}{2} \bar{i}_P(T) \right]}{p}. \quad (6)$$

3 SYNTHETIC $V_{S,app.}(T)$ RESPONSE CURVES

Given the estimation procedure of Section 2 the $V_{S,app.}(T)$ response of simple models can be studied. The synthetic (Z , R) data used are calculated with a propagator matrix approach (e.g. Kennett 1983). This approach includes all multiples, which is important for the syn-

thetic $V_{S,app.}(T)$ estimates to obtain the correct asymptotic behaviour for large T .

Poisson–Birch assumptions, which relate V_P and ρ (density) to V_S (see caption of Fig. 2) are made for the V_S models used in this section, and the slowness is set to $p = 6.0 \text{ s deg}^{-1}$.

3.1 Layer-over-half-space

3.1.1 Examples for four T values

Fig. 1 illustrates the synthetic (Z_{RF} , R_{RF}) estimate for a layer-over-half-space model (Fig. 2a) filtered using four values of the filter-parameter T : 0.5, 2, 8 and 32 s. For each T the resulting $V_{S,app.}(T)$ value is given.

$V_{S,app.}(T = 0.5 \text{ s})$ equals the S velocity of the upper layer, $V_{S,1}$ and $V_{S,app.}(T = 2 \text{ s})$ lies between the S velocity of the upper layer and that of the half-space, $V_{S,1}$ and $V_{S,2}$, respectively. $V_{S,app.}(T = 8 \text{ s})$ is higher than both $V_{S,1}$ and $V_{S,2}$; $V_{S,app.}(T = 32 \text{ s})$ is quite close to $V_{S,2}$.

These differences in estimated $V_{S,app.}(T)$ are caused by converted arrivals (primary and multiples) present on R_{RF} after P at $t = 0$; successively, more of these arrivals interfere with P at $t = 0$ as T is increased.

3.1.2 The basic $V_{S,app.}(T)$ curve

The black curve in Fig. 2(b) illustrates the $V_{S,app.}(T)$ response of the model in Fig. 2(a) calculated from the synthetic receiver functions using eqs (5) and (6) for a suite of logarithmically distributed T values. The four results of Fig. 1 for the same model are marked by small circles. Notice the position of the marker at $T = 8 \text{ s}$ on the part of the $V_{S,app.}(T)$ curve that ‘overshoots’ even $V_{S,2}$.

We observe the following fundamental properties of the basic layer-over-half-space $V_{S,app.}(T)$ response curve in Fig. 2(b):

- (i) $V_{S,app.}(T) = V_{S,1}$ for $T < t_{Ps}$, where t_{Ps} is the delay-time of the P -to- S converted phase (Ps) from the layer boundary. Only the direct P contributes to $R_{RF}(t = 0)$.
- (ii) $V_{S,app.}(T) > V_{S,1}$ for $T > t_{Ps}$, because Ps and possibly several of the multiples interfere with P at $R_{RF}(t = 0)$.
- (iii) $V_{S,app.}(T) > V_{S,2}$ for $T \sim 7t_{Ps}$, where predominantly the strong positive phases Ps and $PpPs$ interfere with P at $R_{RF}(t = 0)$.
- (iv) $V_{S,app.}(T) \rightarrow V_{S,2}$ for large T , because the positive (Ps and $PpPs$) and the negative ($PpSs$ and $PsSs$) main converted phases as well as progressively more of the higher order multiples interfere with P at $R_{RF}(t = 0)$.

The asymptotic convergence to $V_{S,2}$ for large T (property iv) can be understood physically by considering the limit where the wavelength relative to the thickness of the upper layer goes to infinity. In this limit only the parameters of the half-space can play a role. For T above ca. 20 times t_{Ps} , $V_{S,app.}(T)$ is generally converged to within 2 per cent of $V_{S,2}$. However, estimates for such large T are not needed to constrain the asymptotic V_S value by inversion (see Section 5).

3.2 $V_{S,app.}(T)$ for multiple layers

Fig. 2(c) shows a crust-like model with a sediment layer and crystalline crust on a mantle half-space. The $V_{S,app.}(T)$ response of this model is shown in Fig. 2(d) as the black curve.

Qualitatively, each boundary below the free surface produces a $V_{S,app.}(T)$ response like the basic one for a layer-over-half-space

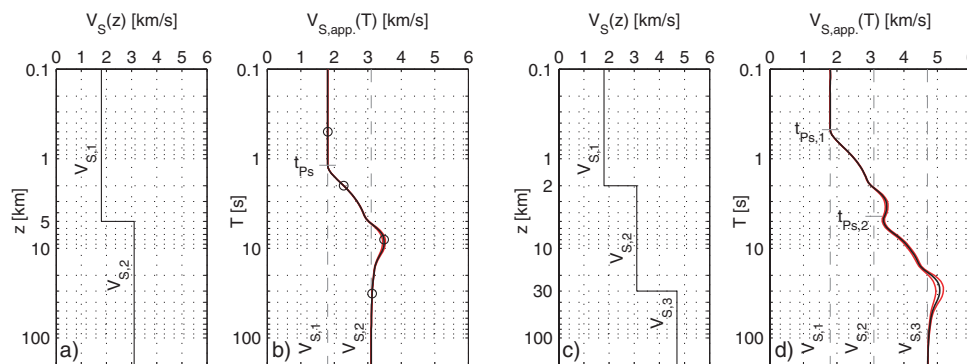


Figure 2. (a) Illustrates the V_S model used to calculate the low-pass filtered receiver functions of Fig. 1 and the black $V_{S,app.}(T)$ curve in (b). The four $V_{S,app.}(T)$ estimates of Fig. 1 are marked with small circles. $T = t_{Ps}$ for the interface, at 1.2 s, is marked as the point where the $V_{S,app.}(T)$ curve bends away from $V_{S,1}$. Notice that the $V_{S,app.}(T)$ curve overshoots $V_{S,2}$ with a maximum at 8.5 s or *ca.* $7 t_{Ps}$. (c) illustrates a crust-like model with 2 km of sediments, 28 km of crystalline crust and a mantle half-space. (d) illustrates the $V_{S,app.}(T)$ response of this model, with t_{Ps} for the two interfaces marked at 0.5 and 4.4 s, respectively. At these values of T the basic response curves as in (b) would bend away from the S velocity of the layer above the interface. However, the $V_{S,app.}(T)$ curve does not fully converge to $V_{S,2}$ before bending off towards $V_{S,3}$ at $T = 4.4$ s, t_{Ps} for the lower interface. Poisson–Birch assumptions, $V_P/V_S = \sqrt{3}$ and $\rho = 320 V_P + 770$, are made for both the model in (a) and (c). The slowness for the black $V_{S,app.}(T)$ curves equals $p = 6.0 \text{ s deg}^{-1}$, but for comparison also $V_{S,app.}(T)$ curves for the extreme slowness values $p = 4.4$ and 7.6 s deg^{-1} are included as red curves.

(Fig. 2b). In general, the response of multilayered models is well approximated as the superposition of such layer-over-half-space part-responses. For each boundary the part-response is shifted along the logarithmic T -axis by multiplication with a factor related to the total t_{Ps} for that boundary and scaled in the $V_{S,app.}$ axis to match the velocity contrast. Thus interfaces result in kinks where T equal the interface t_{Ps} and bumps where $T \sim 7t_{Ps}$ in an observed $V_{S,app.}(T)$ curve.

These scaling properties make it natural to plot $V_{S,app.}(T)$ curves on logarithmic T axes and the models on logarithmic z axes.

3.3 Averaging $V_{S,app.}(T)$ for different slownesses

The slownesses (p) of the observed teleseismic events used in this article (Section 4) are accurately estimated using the global reference model IASP91 (Kennett & Engdahl 1991) and vary from 4.4 to 7.6 s deg^{-1} . The influence of this variation in p on the synthetic $V_{S,app.}(T)$ response curves of Figs 2(b) and (d) is exemplified in these figures as the red curves which represent the $V_{S,app.}(T)$ responses for the two extreme values of p . For both Figs 2(b) and (d) the average of the red curves coincides within drawing accuracy with the black $V_{S,app.}(T)$ curve which is calculated for $p = 6.0 \text{ s deg}^{-1}$, the average slowness of the two extremes. In general it is a good approximation to model the average $V_{S,app.}(T)$ for several events of different slowness by $V_{S,app.}(T)$ computed for the average slowness, that is,

$$\frac{1}{N} \sum_{i=1}^N V_{S,app.}(T, p_i) \cong V_{S,app.}\left(T, \frac{1}{N} \sum_{i=1}^N p_i\right). \quad (7)$$

This does not apply to direct or inverse modelling of receiver functions, where only narrow bins of slowness are typically averaged.

4 APPLICATION TO OBSERVED DATA

The synthetic examples of Section 3 clearly illustrate that $V_{S,app.}(T)$ provides useful information about the absolute S -velocity levels beneath a single station. To test performance of the method on observed data we estimate $V_{S,app.}(T)$ curves for three different broad-band stations: Fürstfeldbruck (FUR), Black Forest Observatory (BFO) and Summit (SUM). These stations represent three very different

geologic settings which are well understood and previously studied in detail using a variety of different geophysical methods. Hence the scope of this section is mainly a validation of the $V_{S,app.}(T)$ method.

4.1 Station settings

FUR is situated just north of the Alps on relatively thick deposits of Molasse overlain by moraine. BFO is on bedrock in the mountains east of the Rhine Graben. Both stations are part of the German Regional Seismic Network (GRSN). SUM is installed centrally on the thick Greenland ice sheet by the GLATIS project (Dahl-Jensen *et al.* 2003) and is now a semi-permanent GEOFON station renamed to SUMG.

4.2 Observed $V_{S,app.}(T)$

Teleseismic events between 50° and 100° were used. Figs 3(a)–(c) illustrate the $V_{S,app.}(T)$ estimates calculated using eqs (5) and (6) from the (Z_{RF} , R_{RF}) estimates of these events. Both ambient noise and deconvolution noise cause a very conspicuous scatter. To subdue outliers, only estimates within the 68 per cent fraction closest to the median (black curve) at each T are plotted as light grey dots. For Gaussian errors this 68 per cent fraction equals one standard deviation and the median equals the mean. For non-Gaussian errors the median is a more robust estimate, although the difference is minimal in most cases.

4.2.1 FUR (109 events)

The $V_{S,app.}(T)$ equals *ca.* 1.0 km s^{-1} for the lowest T values and increases gradually to 4 km s^{-1} at $T \approx 4 \text{ s}$ where the curve forms a bump. A second bump at $T \approx 25 \text{ s}$ reaches *ca.* 4.7 km s^{-1} .

The initial and gradually increasing velocity level of $V_{S,app.}(T)$ indicates soft sediments compacting with depth. The first bump indicates overshoot from a sharp transition to the basement of higher S velocity and the second bump indicates overshoot from Moho thus revealing the crust–mantle transition.

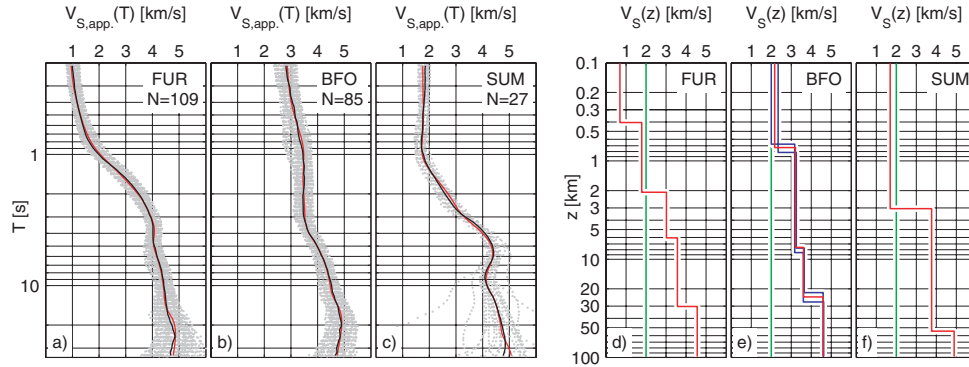


Figure 3. (a)–(c) show $V_{S,app.}(T)$ estimates for the three stations. For each T the light grey dots mark the 68 per cent fraction of the estimates closest to median, shown as the black curve. The red curve is the response of the final inversion model. (d)–(f) illustrate the results of the inversion for each station. The $V_S(z)$ starting models are shown as green curves and the final best-fitting models as red curves. A constant S velocity of 2 km s^{-1} was chosen as starting model. In (e) two additional blue curves illustrate inversion results for BFO using constant V_P/V_S ratios of 1.6 and 1.9, respectively.

4.2.2 BFO (85 events)

For BFO the $V_{S,app.}(T)$ curve starts at 2.8 km s^{-1} and increases smoothly to a moderate bump at $V_{S,app.}(T) \approx 3.5 \text{ km s}^{-1}$ for T in the range of 1–3 s. Hereafter $V_{S,app.}(T)$ increases more steeply and forms a second bump at $V_{S,app.}(T) \approx 5 \text{ km s}^{-1}$ for $T \approx 23 \text{ s}$.

The high initial level of this $V_{S,app.}(T)$ curve indicates crystalline rocks at the surface. The first bump is overshoot from an increase of the S velocity in the uppermost crust and the second bump is overshoot due to Moho.

4.2.3 SUM (27 events)

The $V_{S,app.}(T)$ curve for SUM is almost constant at $1.7\text{--}1.8 \text{ km s}^{-1}$ for $T < ca. 0.9 \text{ s}$. For higher T values the curve forms a sharp bump at $T \approx 5.8 \text{ s}$ reaching 4.4 km s^{-1} . A second and barely resolved maximum is indicated for $T > 30 \text{ s}$, reaching $ca. 5 \text{ km s}^{-1}$.

The constant part of the $V_{S,app.}(T)$ curve at $1.7\text{--}1.8 \text{ km s}^{-1}$ clearly reveals the thick ice sheet with an S velocity of glacial ice (Benjumea & Teixido 2001). After the constant part the $V_{S,app.}(T)$ curve increases steeply, thus revealing a t_{Ps} of $ca. 0.9 \text{ s}$ for the ice-basement transition which equals T at this bend. The first bump is overshoot due to the ice-basement transition, and the second bump is overshoot related to Moho.

5 MODELLING OF $V_S(z)$ FROM $V_{S,app.}(T)$

We apply standard linearized iterative inversion to quantify simple and yet geologically significant models of the S velocity as a function of depth $V_S(z)$.

An observed median $V_{S,app.}(T)$ curve is modelled by a moderate number of horizontal homogeneous layers over a half-space, with sufficient layers to explain the main features of the observed $V_{S,app.}(T)$ curve (see Section 4.2). The inversion is performed using weighted linearized least-squares iteration (e.g. Menke 1989, eq. 9.11)

$$m_{i+1} = m_i + (G^T C_{obs}^{-1} G + C_{mod}^{-1})^{-1} \{G^T C_{obs}^{-1} [d_{obs} - g(m_i)] + C_{mod}^{-1} (m_0 - m_i)\}, \quad (8)$$

where the vector d_{obs} is the observed $V_{S,app.}(T)$ median and the vector m_0 is the prior model of S velocities and interface positions. m_i is the model vector before the iteration step and m_{i+1} is the updated model after the iteration. Relative weighting of data is implemented through C_{obs} which is approximated as a diagonal matrix with elements defined from the 68 per cent fraction centred at the median

(see Fig. 3a–c). C_{mod} is a diagonal matrix with the uncertainties of m_0 which implement possible *a priori* constraints on interval S velocities and interfaces. The interface positions are parametrized in delay time of the Ps phase, t_{Ps} (e.g. Zhu & Kanamori 2000, eq. 2)

$$\Delta t_{Ps} = \Delta z (\sqrt{V_S^{-2} - p^2} - \sqrt{V_P^{-2} - p^2}) \quad (9)$$

and at each iteration the depth vector, z , is updated using the optimized t_{Ps} and V_S parameters in m_i .

The forward mapping, g , represents computation of synthetic wavefields which are deconvolved to form synthetic (Z, R) receiver functions, convolved with the observed average Z_{RF} taken as the basic wavelet and finally inserted in eqs (5) and (6). G is the matrix of partial derivatives computed simply from differences between responses with perturbations of individual parameters. During the iterations G changes slightly.

Note that our inversion is performed without any *a priori* constraints on V_S , that is, the corresponding diagonal elements of C_{mod} are infinite. To exemplify utilization of the high frequency phase arrivals in the receiver function, as for example, Ps from Moho, we have included one pick of a clear Ps phase for each station (see Fig. 4). Such constraints on t_{Ps} for an interface are easily implemented with the parametrization used as one non-infinite diagonal element in C_{mod} .

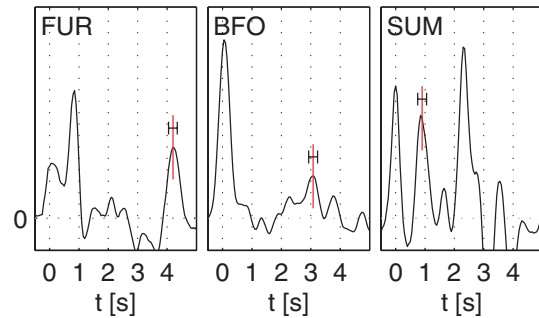


Figure 4. Observed mean R_{RF} with picked Ps phases marked by vertical red lines with error bars attached to show the uncertainty estimate. For FUR and BFO the Moho Ps is picked and for SUM the pick is Ps from the base of the ice. Note the different amplitudes at $t = 0$ for the stations, which directly indicate the absolute V_S level of the near surface: very low at FUR, intermediate at SUM and high at BFO (cf. Section 4.1).

5.1 Inversion results

Figs 3(d)–(f) illustrate the results of the inversion. In the first iteration the interface t_{PS} delay times are fixed and only V_S is optimized. In the following iterations both t_{PS} and V_S are optimized. The final models and their responses (red curves in Fig. 3) are practically independent of the S -velocity starting model. To illustrate this we let the inversion start at a constant S velocity of 2 km s^{-1} (green curves). However, starting at 4 km s^{-1} yields practically the same final models within two to four iterations. This result is in contrast to important previous studies of receiver function inversion where S velocities were constrained by the starting model (Ammon *et al.* 1990; Kind *et al.* 1995). The latter study also used tight constraints on the parameters of the mantle half-space.

The V_P/V_S values were assumed to be 1.8 for sediments and the mantle, 1.73 for the crystalline crust and 2.2 for ice. The effect of these choices on the inferred $V_S(z)$, which is analysed in Section 6, is rather small.

The results of the linearized inversion quantify the significant geological features expected from direct inspection of the three observed $V_{S,app.}(T)$ median curves (Section 4.2).

For FUR the inversion required three interfaces in addition to Moho to reproduce the main features of the observed median $V_{S,app.}(T)$ curve. For BFO only two additional interfaces were needed, whereas for SUM the observed $V_{S,app.}(T)$ median curve was well fitted with only an ice layer, a crystalline crust and a mantle half-space.

In general, our inversion results agree well with models obtained by other studies in the vicinity of FUR, BFO and SUM. For FUR Kind *et al.* (1995) obtained a gradual increase of V_S starting at around 1 km s^{-1} for the near surface increasing to above 3 km s^{-1} at 3 km depth. Our inversion result (Fig. 3d) has two layers with S velocities corresponding to sediments, 0.7 and 1.8 km s^{-1} , respectively. Below the interface at 2.1 km depth V_S increases to above 3 km s^{-1} . This result agrees well with the *ca.* 2.5 km of sediments published in Trümpy & Dal Piaz (1992) and in Hurtig *et al.* (1992) for the position of FUR.

For the crystalline crust below FUR Kind *et al.* (1995) obtained an almost constant and quite high V_S of *ca.* 3.8 km s^{-1} and a Moho depth of 32 km . Our results indicate a crust with an interface at 6.1 km , a Moho depth of 30 km and S velocities of 3.0 and 3.6 km s^{-1} , respectively, for the upper and lower crustal layer. The resulting average V_S of the crystalline crust is 3.5 km s^{-1} and the average of the entire crust is 3.4 km s^{-1} . This is consistent with a crustal average V_S of 3.4 – 3.6 km s^{-1} obtained for the region of FUR and BFO using dispersion of Rayleigh waves (Pasyanos & Walter 2002, Fig. 9d). The Moho depth obtained for FUR is also consistent with the 30 – 32 km in the Moho depth map of Ziegler & Dézes (2005).

The Moho depth obtained for BFO is 24 km (Fig. 3e) which compares very well to the 22 – 24 km found by Kind *et al.* (1995) and in the map of Ziegler & Dézes (2005). Our inversion result yields crustal interfaces at 0.7 and 7.6 km which agrees well with interfaces at 1 km and *ca.* 7 km obtained by (Kind *et al.* 1995) who has an additional interface at 14 km which is not needed to explain the $V_{S,app.}(T)$ data. The S velocity of the thin surface layer (2.2 km s^{-1}) matches well with that of Kind *et al.* (1995). For the crustal layers our velocities of 3.2 and 3.6 km s^{-1} are slightly (0.1 – 0.2 km s^{-1}) higher than those of Kind *et al.* (1995). Still the crustal average V_S of 3.5 km s^{-1} we obtain is consistent with the surface wave inversion results of Pasyanos & Walter (2002).

For FUR and BFO we obtain relatively well-constrained upper-mantle S velocities of 4.5 – 4.6 km s^{-1} without applying any *a priori* S velocity information. These values are slightly higher but agree well with the uppermost mantle S velocity of 4.47 km s^{-1} in the global reference model IASP91 (Kennett & Engdahl 1991). Our result is also consistent with uppermost mantle S velocities in southern Germany of 4.3 – 4.6 km s^{-1} obtained by Pasyanos & Walter (2002) and 4.4 – 4.5 km s^{-1} at 40 km depth obtained by Friederich (1998).

The inversion result for SUM (Fig. 3f) yields an ice thickness of 3.1 km with an S velocity of 1.7 km s^{-1} , which is within the range expected for glacial ice (Benjumea & Teixido 2001). The obtained ice thickness is very consistent with the 3053 m obtained in the GISP2 ice core drilling near SUM (Gow *et al.* 1997). The depth of Moho for SUM obtained by Dahl-Jensen *et al.* (2003) is *ca.* 50 km compared to our inversion result of 53 km . The V_S obtained for the crystalline crust is 3.8 km s^{-1} , which is quite well constrained by the high overshoot at T equal to 5 – 6 s . This result is also consistent with a high average crustal V_P of 6.7 km s^{-1} obtained by combined land-sea seismic refraction experiments west of SUM in the Scoresby sund region (Mandler & Jokar 1998) where also large crustal thicknesses (up to 48 km) were obtained. For a V_P/V_S ratio in the range 1.7 – 1.8 a V_S of 3.7 – 3.9 km s^{-1} corresponds to the average V_P of that study.

From inversion of surface wave dispersion curves Darbyshire *et al.* (2004) obtained a relatively high S velocity (up to *ca.* 5 km s^{-1}) in the uppermost mantle (50 – 100 km depth) beneath central Greenland. This unusually high result is consistent with our result of 4.9 km s^{-1} obtained for SUM although our mantle velocity for SUM is not very well constrained due to the larger scatter of $V_{S,app.}(T)$ at T values greater than *ca.* 10 s (see Fig. 3c).

6 ERROR SOURCES AND APPLICATION PERSPECTIVES

The earthquake distances used (Section 4.2) yield minimum time delays between the phases P and PP of *ca.* 115 s , posing an upper limit to the values of T . For T below this limit the PcP phase still might interfere. However, this phase is much weaker than P and typically disregarded in standard receiver function estimation which forms the basis of the $V_{S,app.}(T)$ method.

The assumptions of V_P/V_S ratios in model layers influence the inversion of the $V_{S,app.}(T)$ curves. This error source may be assessed from Fig. 3(e) where the data from BFO were also inverted using a constant V_P/V_S ratio equal to 1.6 and 1.9 (blue curves). Mainly the depths are influenced by the choice of V_P/V_S whereas the difference in the V_S levels are very small. Thus, even for such very large variations in V_P/V_S the S -velocity structure of the crust–mantle is by no means masked. We also note that the RMS residual for the response of the final models obtained using 1.6 and 1.9 are 18 and 25 per cent higher than the residual obtained using the preferred V_P/V_S ratios, although visually their fit were still reasonable. This indicates that to some degree information on V_P/V_S can also be constrained using $V_{S,app.}(T)$. However, inverting for both V_S , depths and V_P/V_S would render the inversion much more non-linear and prone to non-uniqueness.

In $V_{S,app.}(T)$ estimation the best accuracy is expected for broadband stations with many high-quality events. By experience, events below magnitude *ca.* 6.5 are often influenced by long periodic noise (tens to hundreds of seconds) due to their lower signal strength. The level of this noise is quite site dependent and is a main contributor to the large scatter at high T values. However, with many events available the $V_{S,app.}(T)$ median is a very robust estimate.

For temporary stations, the availability of events with broad frequency-information may pose a limitation. With high frequencies available, as for local earthquakes, the near-surface S -velocity stratification may be resolved in detail so that the $V_{S,app.}(T)$ method may also be an important supplement to site-effect modelling in earthquake hazard studies.

7 CONCLUSION

We have presented a novel and simple method to emphasize the information on absolute S velocities present in receiver functions. The method is based on estimation of P -wave angles of surface particle motion from low-pass filtered (Z , R) receiver functions at $t = 0$. These estimates and the P -wave slowness are combined in an expression for the free-surface effect to produce estimates of apparent half-space S velocity as a function of the low-pass (smoothing) filter-parameter, T .

For synthetic data our results show that the $V_{S,app.}(T)$ method recovers the absolute levels of the S velocity from the near-surface to the uppermost mantle beneath a single three-component broad-band station.

The $V_{S,app.}(T)$ method was validated by application to observed data from three different broad-band stations with well studied geological settings. Significant crustal features beneath each of the stations stand out in the observed $V_{S,app.}(T)$ curves, with the absolute level of the S velocities clearly emphasized. The $V_S(z)$ models for the crust and uppermost mantle obtained using linearized inversion quantify the features indicated by visual inspection of the $V_{S,app.}(T)$ curves.

Our inversion results, which were obtained without prior constraints on V_S , are in good agreement with independent results obtained using other methods. These methods include drilling, seismic reflection/refraction, Rayleigh wave dispersion analysis and classical receiver function modelling/inversion using narrow *a priori* velocity constraints.

Thus, we conclude that when a broad frequency-band is preserved in (Z , R) receiver functions, it is, indeed, possible to derive the absolute values of S velocity from the near surface to the uppermost mantle without relying on prior velocity values or supplementary data like surface wave-dispersion. These findings contrast previous influential studies of receiver function inversion where S velocities were constrained by starting models and *a priori* mantle parameters.

We propose the computation of $V_{S,app.}(T)$ curves for individual three-component broad-band stations for several purposes:

- (i) A direct indicator of the absolute S -velocity levels at depth
- (ii) Data for inverse modelling of S -velocity stratification
- (iii) A supplementary constraint to reduce the non-uniqueness of conventional receiver function analysis/inversion

Matlab code for computation of $V_{S,app.}(T)$ curves is available upon request.

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